Exploring the QCD phase diagram with fluctuations of conserved charges

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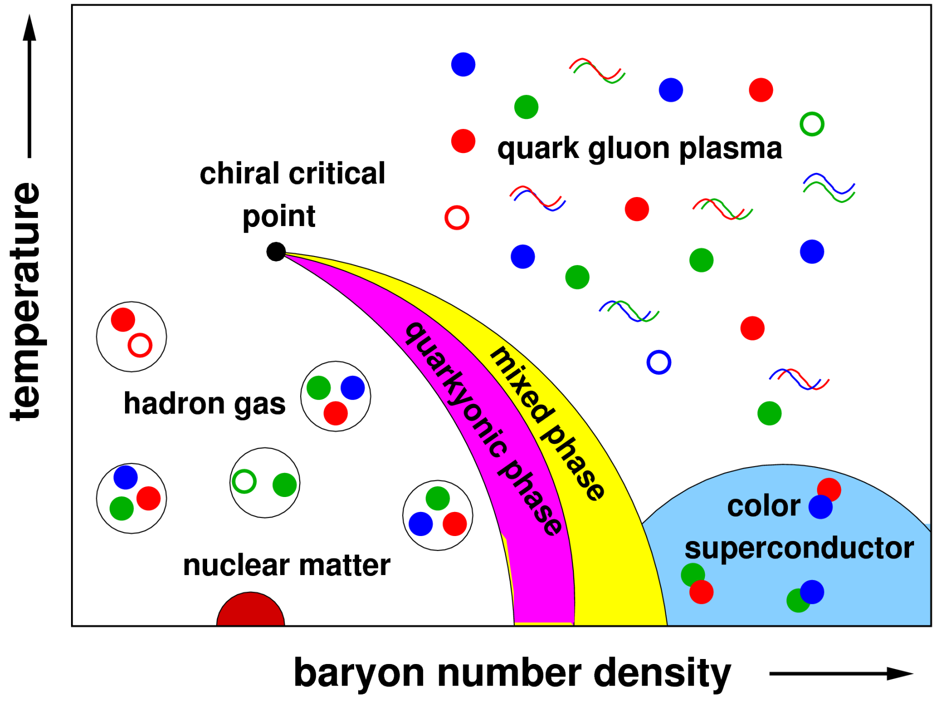


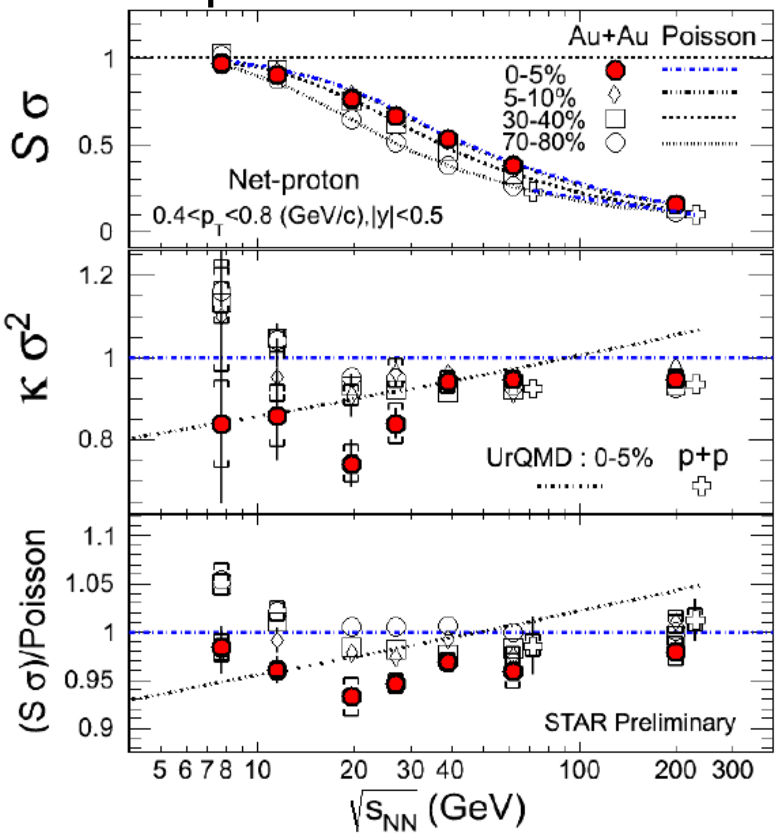
OUTLINE

- Universal (generic?) features of higher order cumulants – insight from O(4) scaling
- A dip in the kurtosis – a signal for the critical point?
- Conserved charge fluctuations and freeze-out (more details: S. Mukherjee)

Motivation

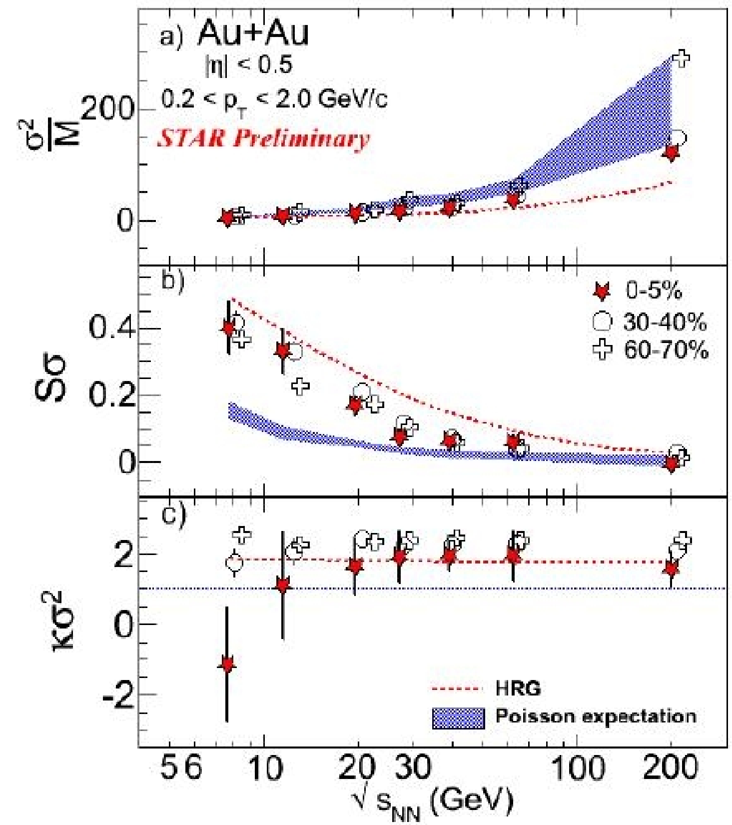
STAR data on net proton number and electric charge fluctuations

net proton number



X. Luo (STAR), Quark Matter 2012

net electric charge



L. Kumar (STAR), Quark Matter 2012

many questions: deviations from HRG just because HRG .ne. QCD or more profound, i.e. equilibrium, grand-canonical approach invalid? Where are the large effects "predicted" by models? negative kurtosis and/or 6th order cumulants? dip in skewness and kurtosis? ....

Bulk thermodynamics and response functions

– probing the response of a thermal medium to an external field, i.e. variation of one of its external control parameters: $T, \ \mu, \ m_q$

(generalized) response functions == (generalized) susceptibilities

pressure: $$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}, m_{u,d,s})$$

energy density

$$\frac{\epsilon}{T^4} = \frac{1}{VT^2} \frac{\partial \ln Z}{\partial T}$$

particle number density

$$\frac{n_q}{T^3} = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial \mu_q / T}$$

order parameter

$$\frac{\langle \bar{\psi}\psi \rangle}{T^3} = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial m_q / T}$$

thermal fluctuations

density fluctuations

condensate fluctuations

generalized susceptibilities: $$\boxed{\frac{\partial^{i+j+k} p/T^4}{\partial T^i \partial \hat{\mu}_X^j \partial \hat{m}_q^k}}$$

$$\hat{A} \equiv A/T$$

Susceptibilities

– probing the response of a thermal medium to an external field, i.e. variation of one of its external control parameters: T, μ, m_q

(generalized) response functions == (generalized) susceptibilities

pressure: $$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}, m_{u,d,s})$$

particle number density
$$\frac{n_q}{T^3} = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial \mu_q / T}$$

quark number susceptibility
$$\chi_q = \frac{\partial\, n_q / T^3}{\partial \mu_q / T}$$

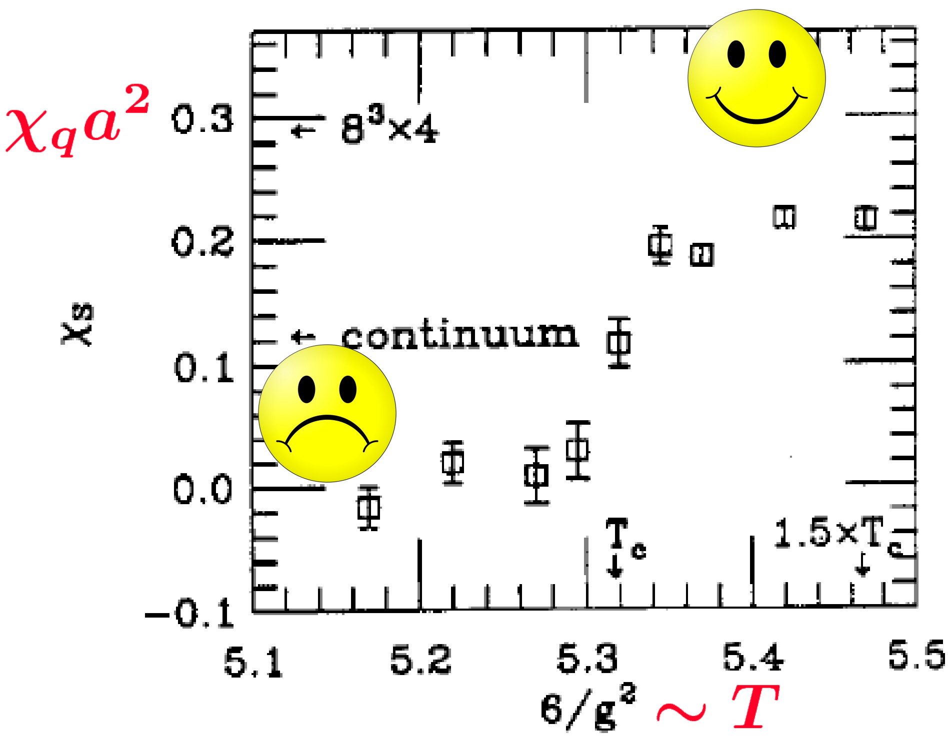
4th order cumulant
$$\chi_4^q = \frac{1}{VT^3} \frac{\partial^4 \ln Z}{\partial (\mu_q / T)^4}$$

generalized quark number susceptibilities:
$$\boxed{\frac{\partial^{i+j+k} p / T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}}$$

$$\hat{\mu}_X \equiv \mu_X / T$$

Quark number Susceptibility – the 25th anniversary

susceptibility = the quality of being susceptible



quark number susceptibility = response of the quark number density to an infinitesimal chemical potential (external field)

Is the medium receptive to a non-zero quark number density?

confinement;
chiral symmetry breaking

deconfinement;
chiral symmetry restoration

$$n_q = \frac{1}{V}\frac{\partial \ln Z}{\partial \mu_q/T}$$

$$\chi_q = \frac{\partial\, n_q}{\partial \mu_q}$$

S.A. Gottlieb, W. Liu, D. Toussaint, R.L. Renken and R.L. Sugar, Phys. Rev. Lett. 59, 2247 (1987)

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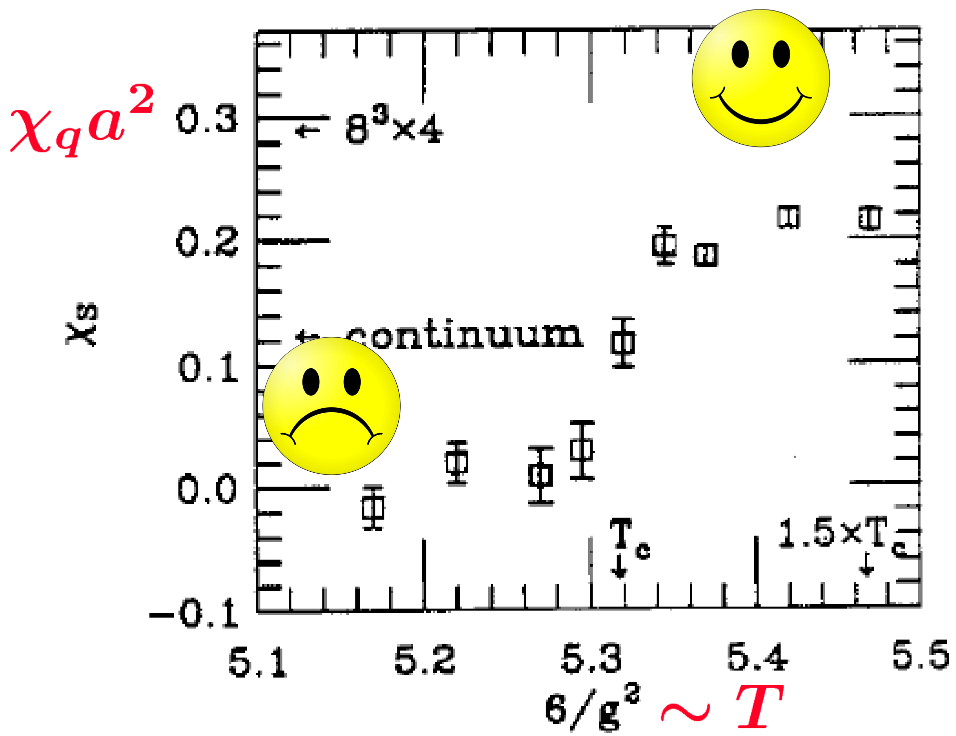


quarks like to be in the QGP !!

Are they the carrier of conserved charges in the QGP or are there other more relevant d.o.f. around?

di-quark bound states in the QGP as source for the sQGP?

E. Shuryak, I. Zahed,
Phys. Rev. D 70, 054507 (2004)

confinement;
chiral symmetry breaking

deconfinement;
chiral symmetry restoration

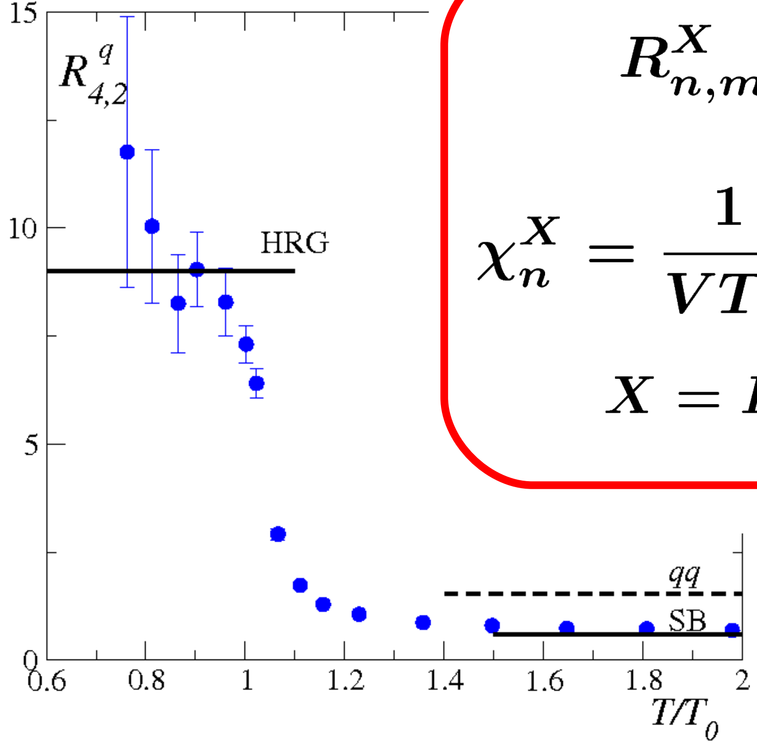
higher order cumulants can rule out such a scenario

S.A. Gottlieb, W. Liu, D. Toussaint, R.L. Renken and R.L. Sugar, Phys. Rev. Lett. 59, 2247 (1987)

Ratios of 4th and 2nd order net charge cumulants

ratios of net quark (baryon) number and net electric charge cumulants rule out a large di-quark contribution in the QGP

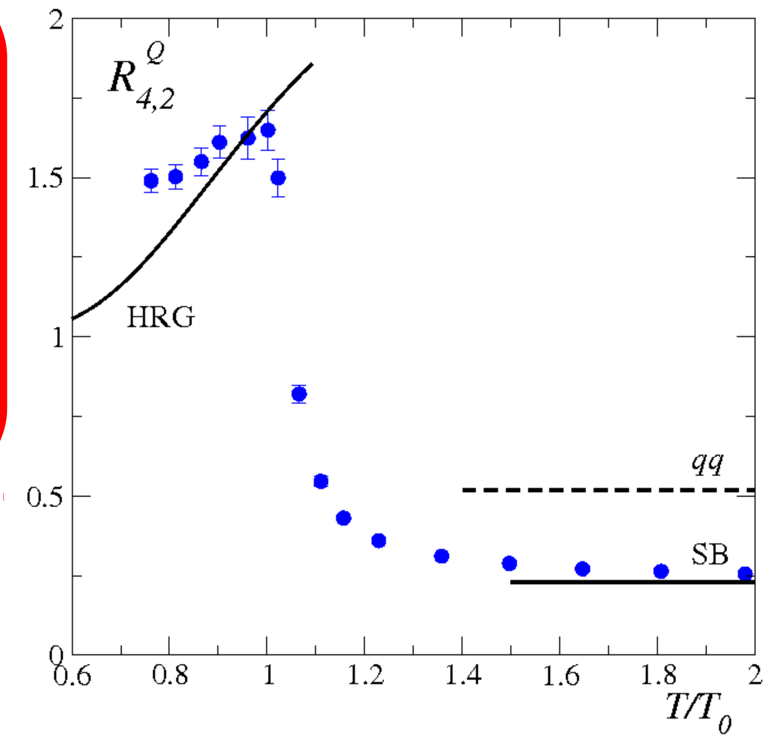
S. Ejiri, FK and K. Redlich, Phys. Lett. B 633, 275 (2006)



$$R^X_{n,m} = \frac{\chi^X_n}{\chi^X_m}$$

$$\chi^X_n = \frac{1}{VT^3}\frac{\partial^n \ln Z}{\partial(\mu_q/T)^n}$$

$$X = B, Q, S, q, \ldots$$



cumulant ratios are sensitive to the thermal properties (relevant d.o.f.) of a strongly interacting, thermal medium

Bulk Thermodynamics and Critical Behavior

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{1+1/\delta} f_s(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

- critical behavior controlled by two relevant fields: t, h
  - all couplings that do not explicitly break chiral symmetry contribute in leading order only to 't', e.g., $T,\ \mu_B,\ \mu_Q,\ \mu_S, ...$

$$t = \frac{1}{t_0}\left(\left(\frac{T}{T_c} - 1\right) - \kappa_B \left[\left(\frac{\mu_B}{T}\right)^2 - \left(\frac{\mu_B^c}{T}\right)^2\right]\right)$$

$$\boxed{h = \frac{1}{h_0}\frac{m_l}{m_s}}$$

control parameter for amount of chiral symmetry breaking

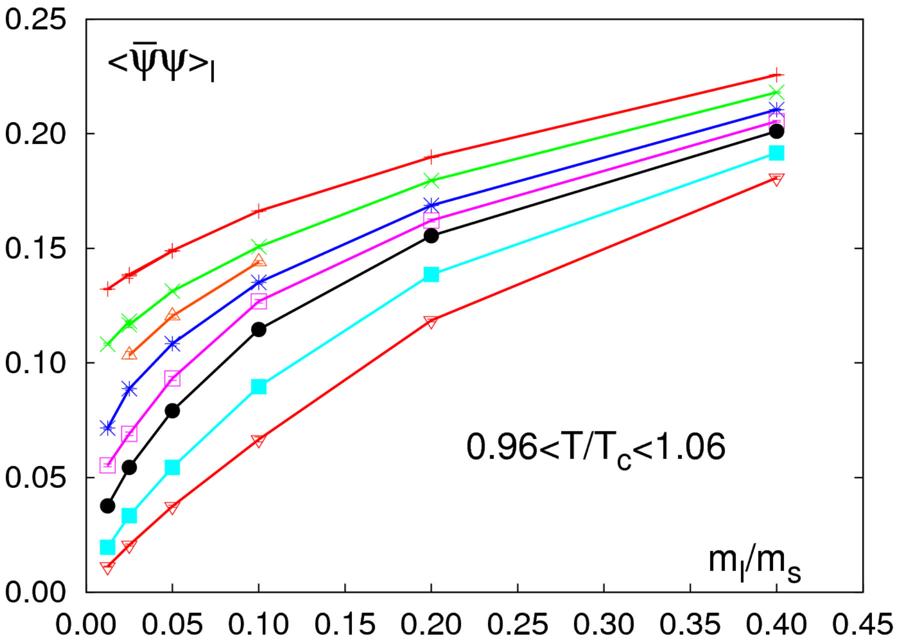
K. G. Wilson, Nobel prize, 1982

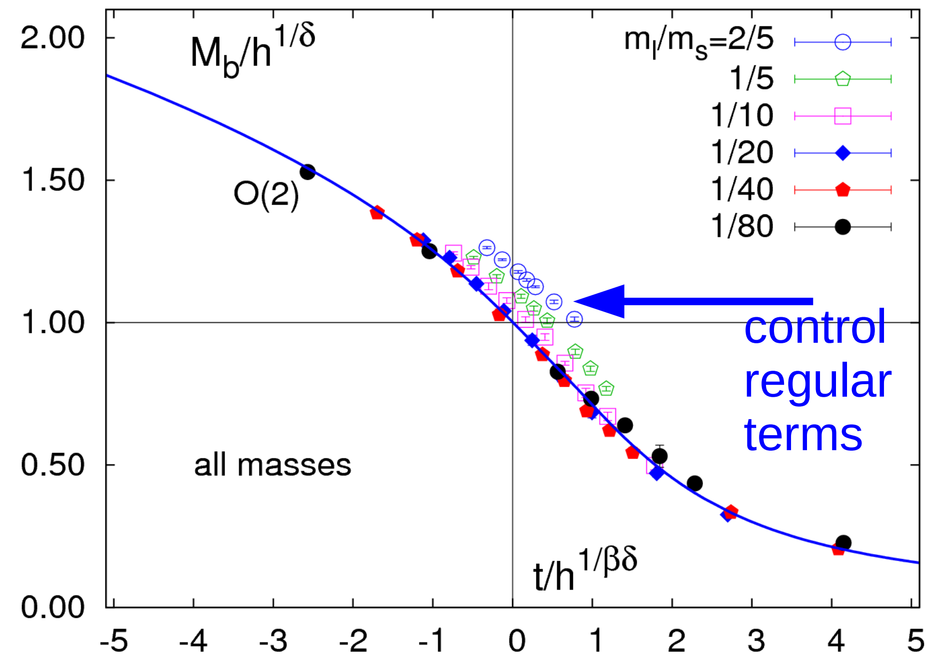
non-universal scales $T_c,\ \kappa_B,\ t_0,\ h_0$

O(4) Scaling in QCD: (I) the order parameter

magnetic equation of state: $M_b = h^{1/\delta} f_G(z)$ + regular

$$\boxed{M_b \equiv \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} \quad , \quad z \equiv t/h^{1/\beta\delta}}$$



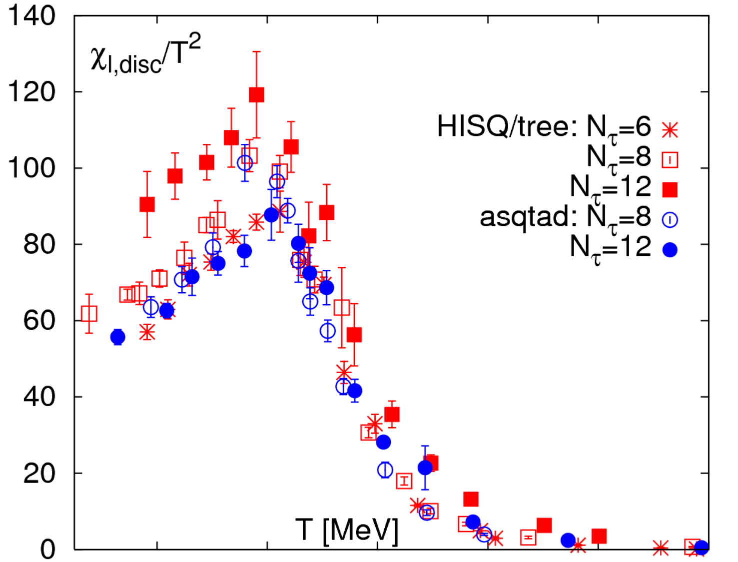


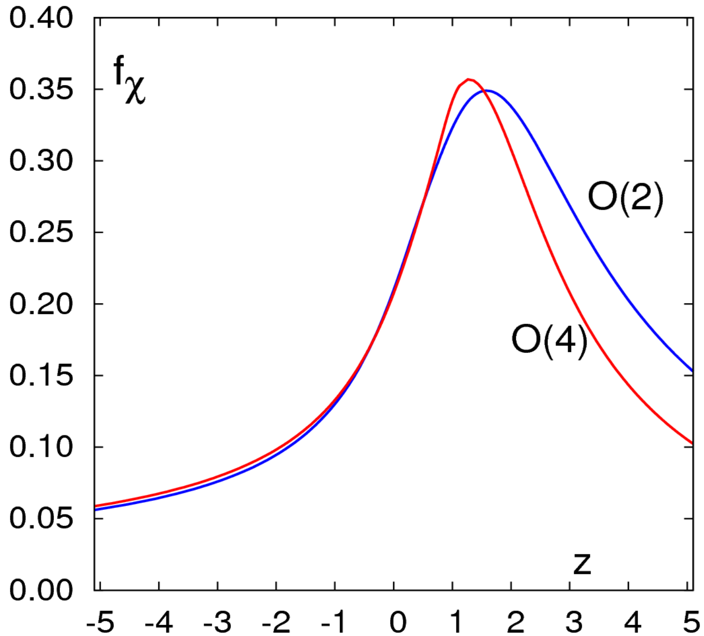
p4-action: $N_\sigma^3 \times 4$, $N_\sigma = 16,\ 32$, $400\mathrm{MeV} \lesssim m_{ps} \lesssim 75\mathrm{MeV}$

S. Ejiri et al (BNL-Bielefeld), Phys. Rev. D80, 094505 (2009)

this fixes $T_c,\ t_0,\ h_0$

O(4) Scaling in QCD: (II) Chiral Transition Temperature





– locate pseudo-critical temperature from chiral susceptibility

$$\chi_{m,l}(T) = \frac{\partial\langle\bar\psi\psi\rangle}{\partial m_l}$$
$$= \chi_{l,disc} + \chi_{l,con}$$

– peak location is controlled by a universal scaling function

$$\frac{m_s^2\chi_{m,l}}{T^4} = \left(\frac{1}{h_0}h^{1/\delta-1}f_\chi(z) + regular\right)$$

$$z_{max} = 1.33(5)$$

$$\frac{T_c(m_q)-T_c(0)}{T_c(0)} = \frac{1}{z_0 z_{max}}\left(\frac{m_l}{m_s}\right)^{1/\beta\delta} + reg.$$

O(4) Scaling in QCD: (III) Curvature of the critical line

Bielefeld-BNL, Phys. Rev. D83, 014504 (2011)

- "thermal" fluctuations of the order parameter

$$t \equiv \frac{1}{t_0}\left(\left(\frac{T}{T_c}-1\right)-\kappa_q\left(\frac{\mu_q}{T}\right)^2\right)\;,\; z = t/h^{1/\beta\delta}$$

$$M_b \equiv \frac{m_s\langle\bar{\psi}\psi\rangle}{T^4} = h^{1/\delta} f_G(z)$$

scaling function of order parameter

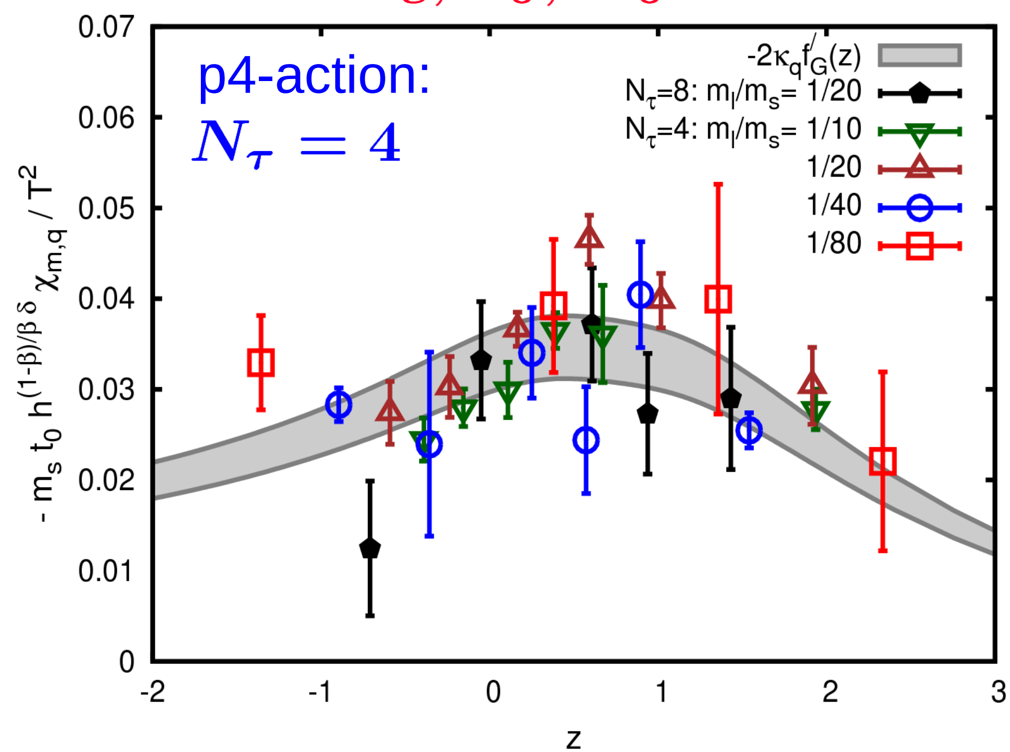
fixes non-universal parameter

$$T_c,\; t_0,\; h_0$$

$$\frac{\chi_{m,q}}{T} = \frac{\partial^2\langle\bar{\psi}\psi\rangle/T^3}{\partial(\mu_q/T)^2}$$

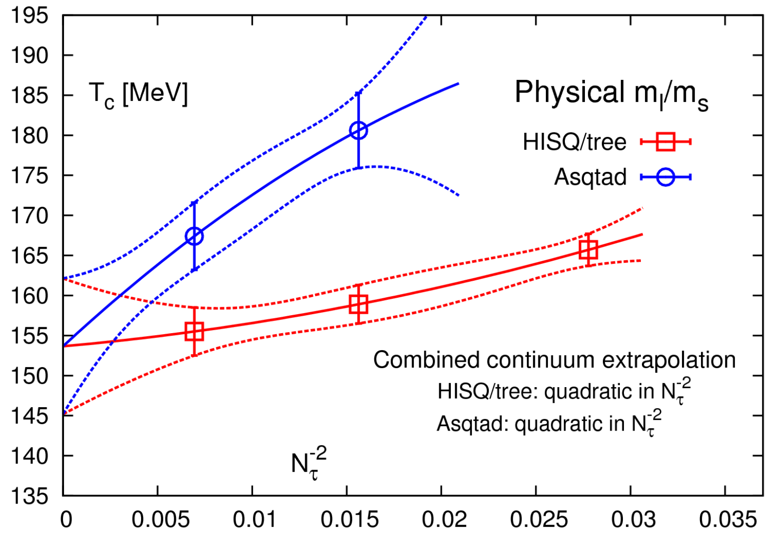
$$= \frac{2\kappa_q T}{t_0 m_s} h^{(\beta-1)/\delta\beta} f'_G(z)$$

$$\boxed{\kappa_B = 0.0066(7)}$$



Crossover temperature at and close to $\mu_B = 0$

The transition temperature at vanishing chemical potential:



crossover identified by peak in the chiral susceptibility:

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

A. Bazavov et al (HotQCD Collaboration), Phys. Rev. D 85, 054503 (2012)

consistent with transition temperatures determined by the Budapest-Wuppertal collab.

Y. Aoki et al., JHEP 0906 (2009) 088

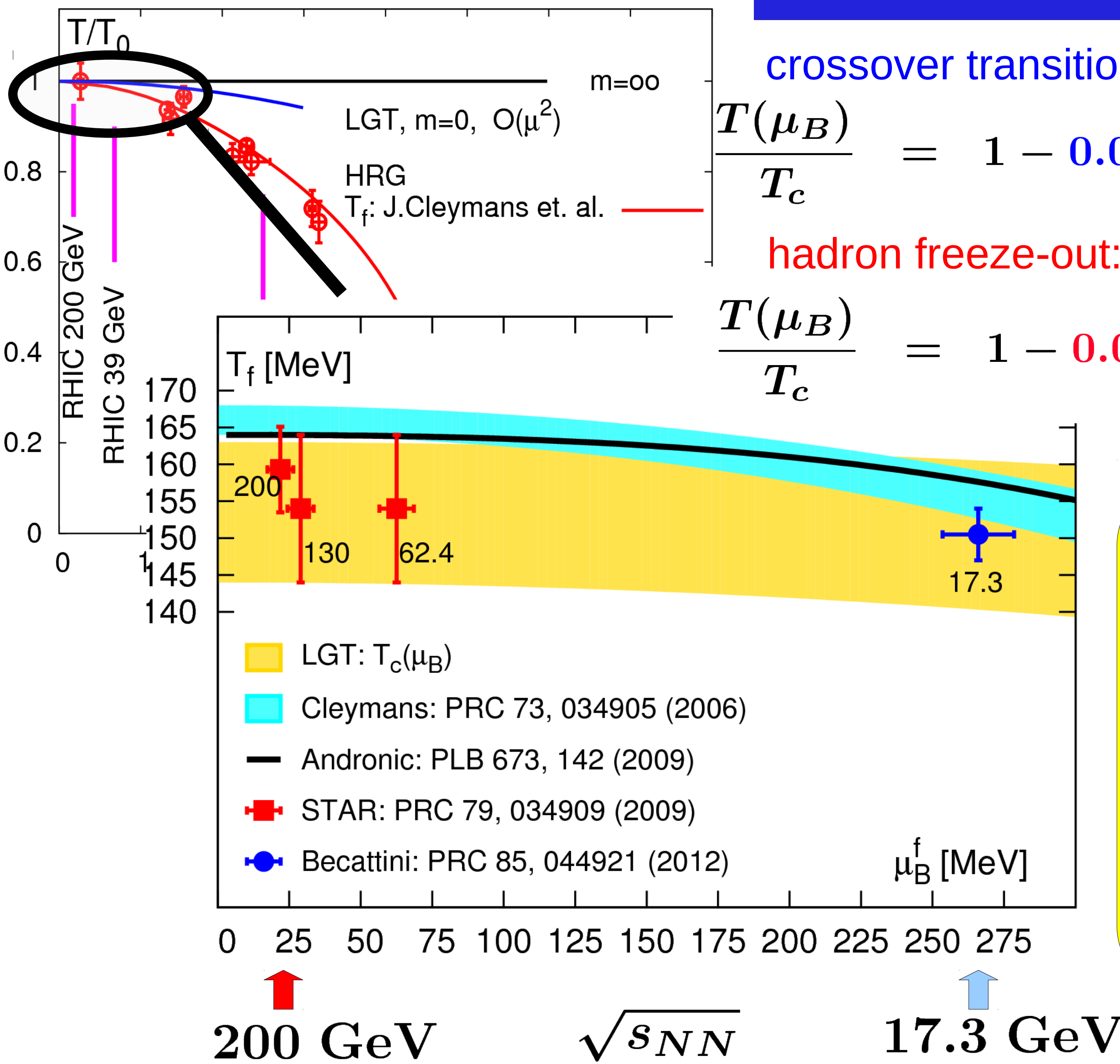
Curvature of the transition line for small μ_B :

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - 0.0066(7)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

Bielefeld-BNL, Phys. Rev. D 83, 014504 (2011)

similar: G. Endrodi et al., JHEP 1104, 001 (2011)

Chiral Transition and Freeze-out



crossover transition:

$$\frac{T(\mu_B)}{T_c} = 1 - 0.0066(7)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

hadron freeze-out:

$$\frac{T(\mu_B)}{T_c} = 1 - 0.023\left(\frac{\mu_B}{T}\right)^2 - c\left(\frac{\mu_B}{T}\right)^4$$

phenomenological freeze-out curve, QCD transition line and experimental data (obtained by assuming the validity of the HRG model) are consistent for

$$\mu_B/T \lesssim 2$$

Critical behavior and higher order cumulants

pressure: $$\frac{p}{T^4} = -h^{1+1/\delta} f_s(\underbrace{t/h^{1/\beta\delta}}_{z}) - f_r(V,T,\vec{\mu})$$

$$1 + 1/\delta = (2-\alpha)/\Delta \; , \; \Delta \equiv \beta\delta$$

$O(4): \; \alpha = -0.213$

$Z(2): \; \alpha = +0.107$

- first divergent susceptibility for n=3 or n=6

$$\chi_{B,\mu_B}^{(n)} \sim \begin{cases} m_q^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(z) \; , \; \mu_B = 0 \\ m_q^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(z) \; , \; \mu_B > 0 \end{cases}$$

$$(f_s(z) \equiv A f_f(z))$$

$$\boxed{t = \frac{1}{t_0}\left(\left(\frac{T}{T_c} - 1\right) - \kappa_B \left[\left(\frac{\mu_B}{T}\right)^2 - \left(\frac{\mu_B^c}{T}\right)^2\right]\right)}$$

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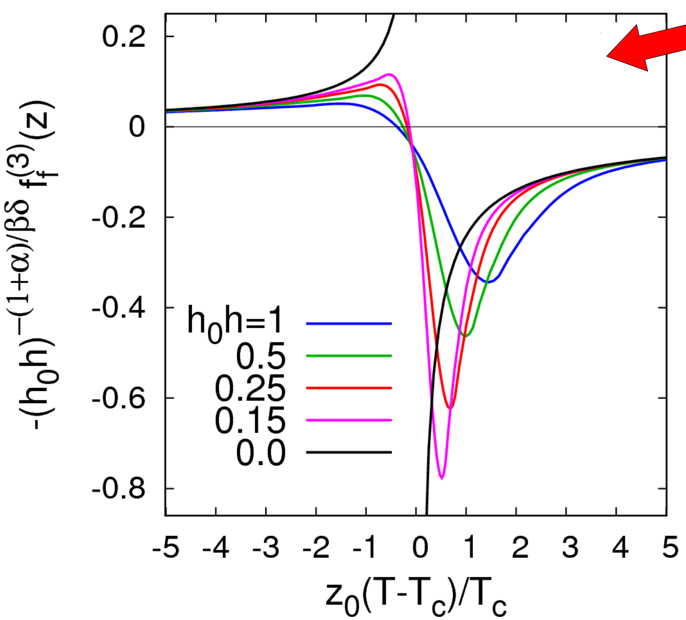
$$(f_s(z) \equiv A f_f(z))$$

$$\chi^{(3)}_{B,\mu},\; \chi^{(6)}_{B,0} \sim f_f'''(z)$$



controlled by third derivative of the singular part of the free energy

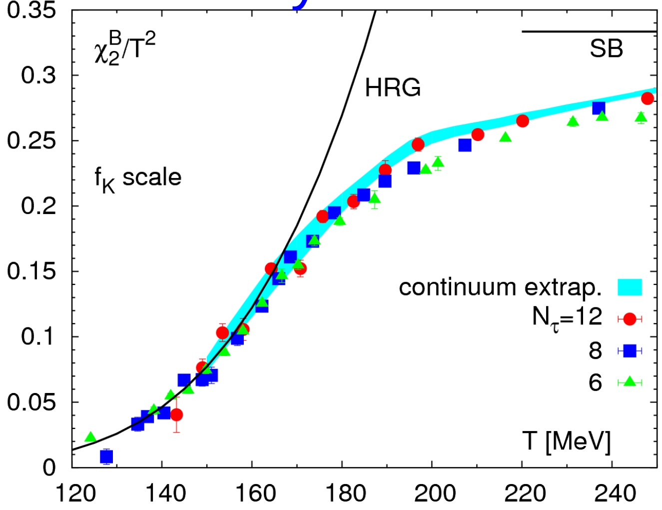
→ universal, O(4) scaling function

B. Friman, FK, K. Redlich, V. Skokov, arXiv:1103.3511

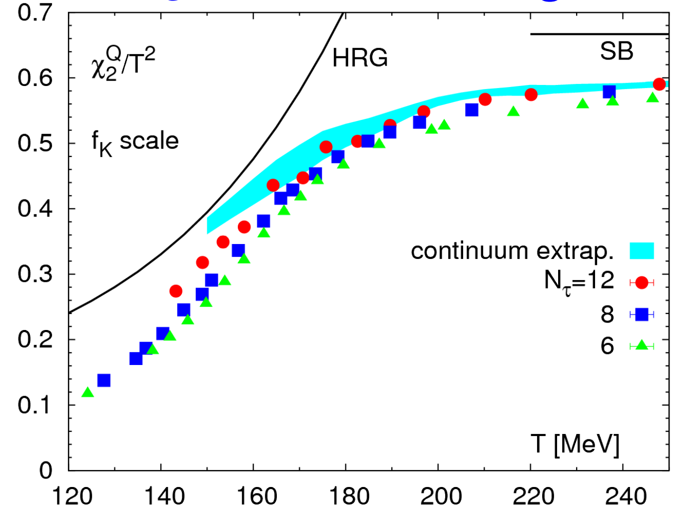
Quadratic charge fluctuations: $\mu_B = 0$

continuum extrapolated results: A. Bazavov et al (hotQCD), arXiv:1203.0784

B: baryon number



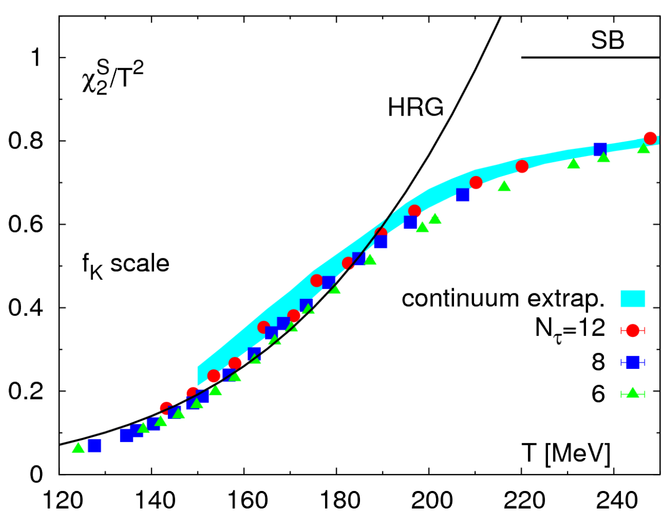
Q: electric charge



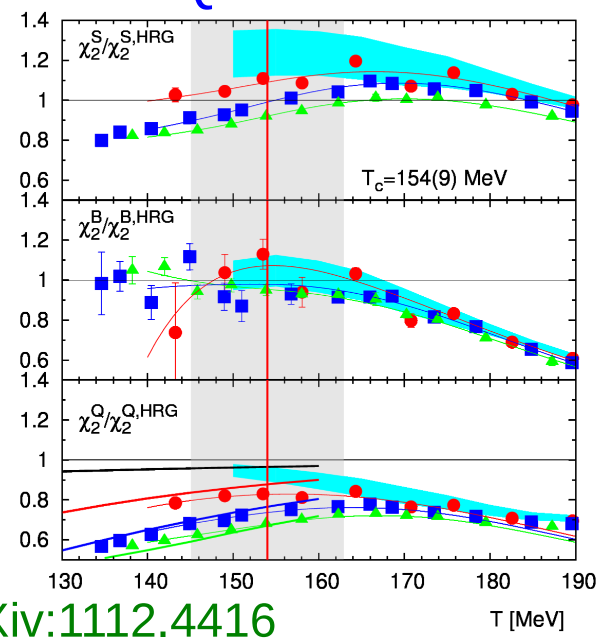
HISQ-action on $(4N_\tau)^3 \times N_\tau$ lattices;

statistics:
~3000 conf./T
~1500 source vec.

S: strangeness



BQS vs. HRG



(2+1)-flavor QCD physical strange quark sector;

$m_l/m_s = 1/20$

$m_{ps} \simeq 160$ MeV

consistent with Wuppertal-Budapest, arXiv:1112.4416

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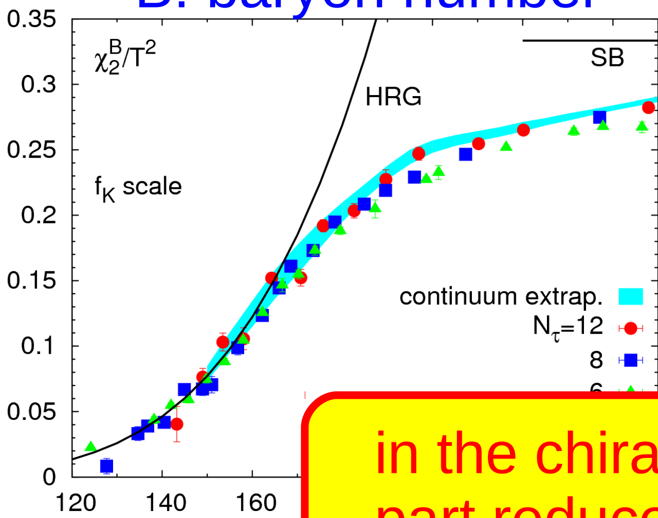

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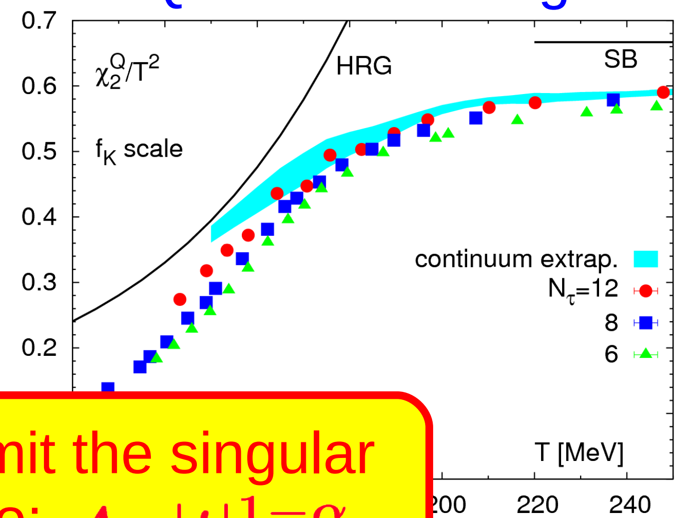
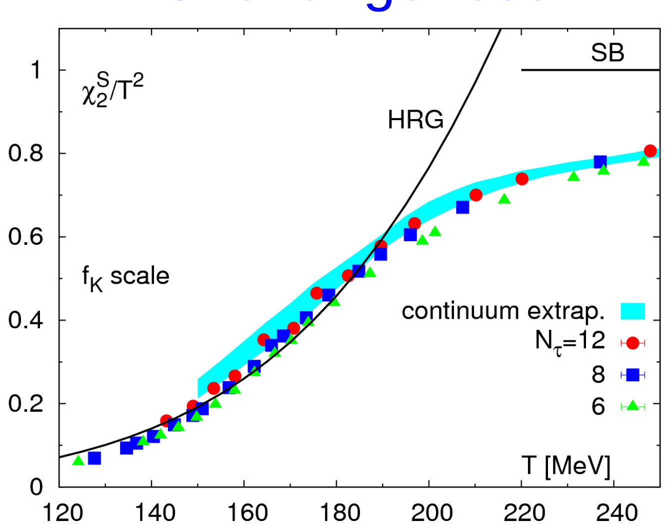
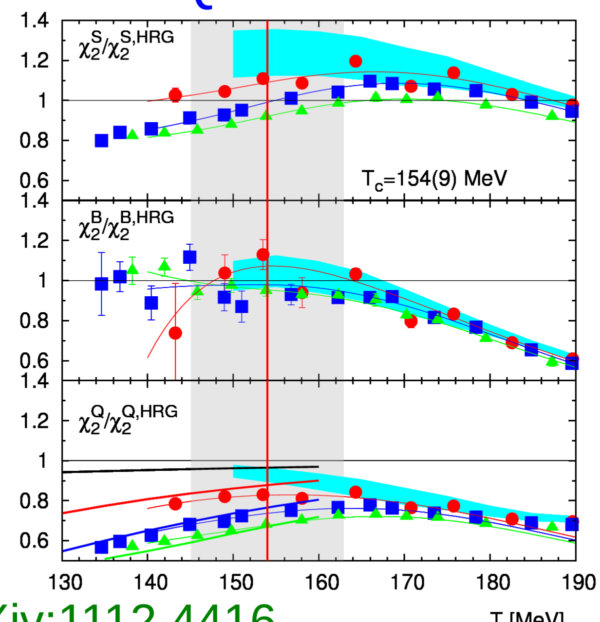


in the chiral limit the singular part reduces to: $A_\pm |t|^{1-\alpha}$

S: str[illegible]eness



[illegible] HRG



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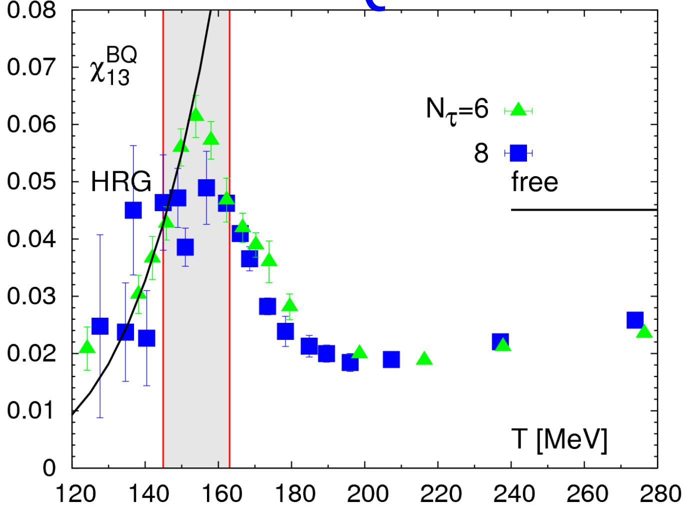
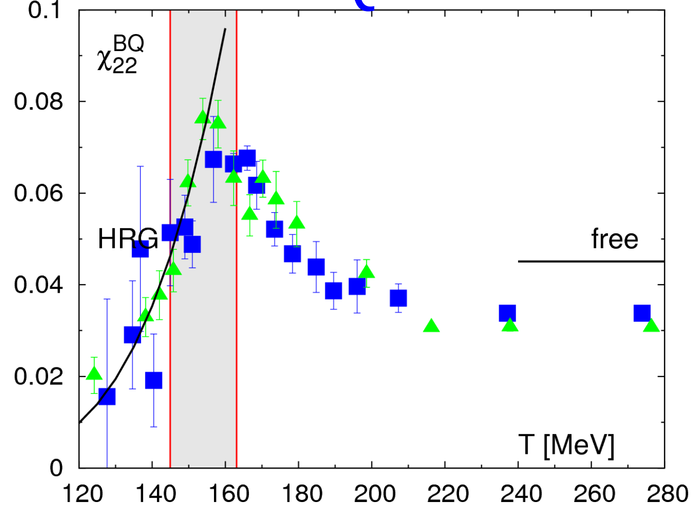
consistent with Wuppertal-Budapest, arXiv:1112.4416

Some 4th order charge fluctuations: $\mu_B = 0$

B1Q3



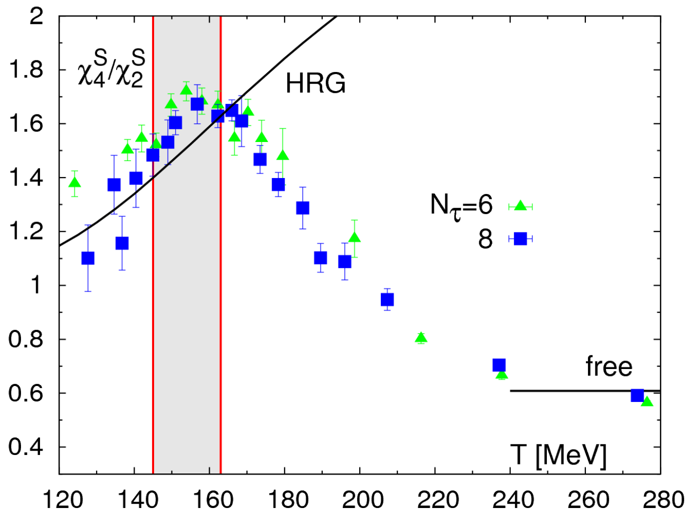
B2Q2



HISQ-action on $(4N_\tau)^3 \times N_\tau$ lattices;

statistics:
~3000 conf./T
~1500 source vec.

$\kappa_S \sigma_S^2$ S4/S2



in the chiral limit the singular part reduces to: $\sim A_\pm |t|^{-\alpha}$

generates cusps in 4th order cumulants at

$$\mu_B = 0$$

band:
$T_c = (154 \pm 9)\mathrm{MeV}$

Bielefeld-BNL, preliminary   similar: S. Borsanyi, Quark Matter 2012

Universal properties of the 6^{th} order cumulant at $\mu_B = 0$

$$\mu_B = 0: \ \chi_{6,0}^B = -(2\kappa_B t_0^{-1})^3 h^{-(1+\alpha)/\Delta} f_s'''(z) + regular$$

in the chiral limit the singular diverges: $\sim A_\pm |t|^{-(1+\alpha)}$

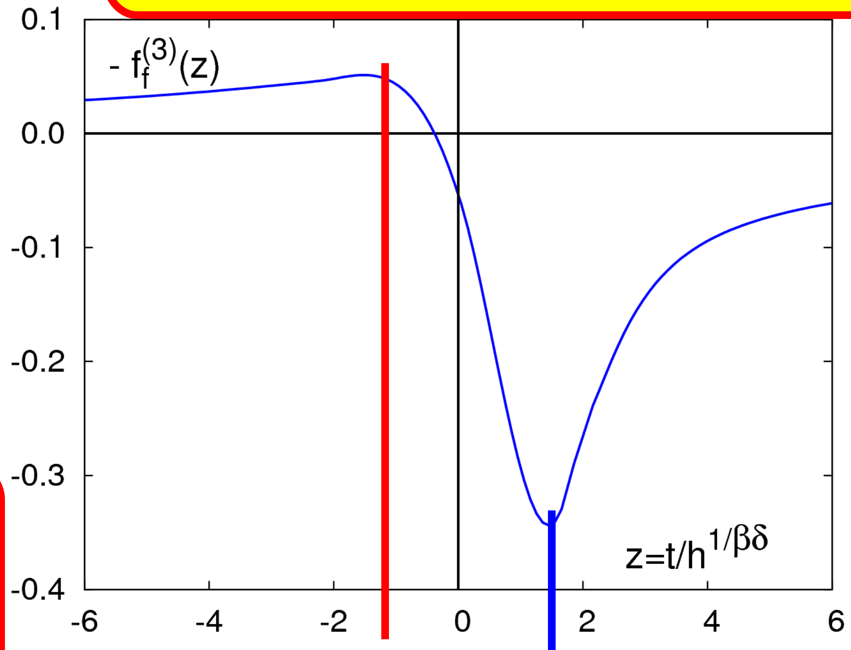
the width of the transition region

(as seen by χ_6^B)

$$\Delta z = z_+ - z_- = (t_+ - t_-)/h^{1/\beta\delta}$$

universal numbers

$$\Delta z = z_+ - z_- \simeq 3$$



$z_- \simeq -1.50$ $z_+ \simeq 1.48$

$$T_+ - T_- = \frac{1}{\Delta z} \frac{t_0 T_c}{h_0^{1/\beta\delta}} \left(\frac{m_l}{m_s}\right)^{1/\beta\delta}$$

$$T_+ - T_- \simeq 0.20(5) T_c$$

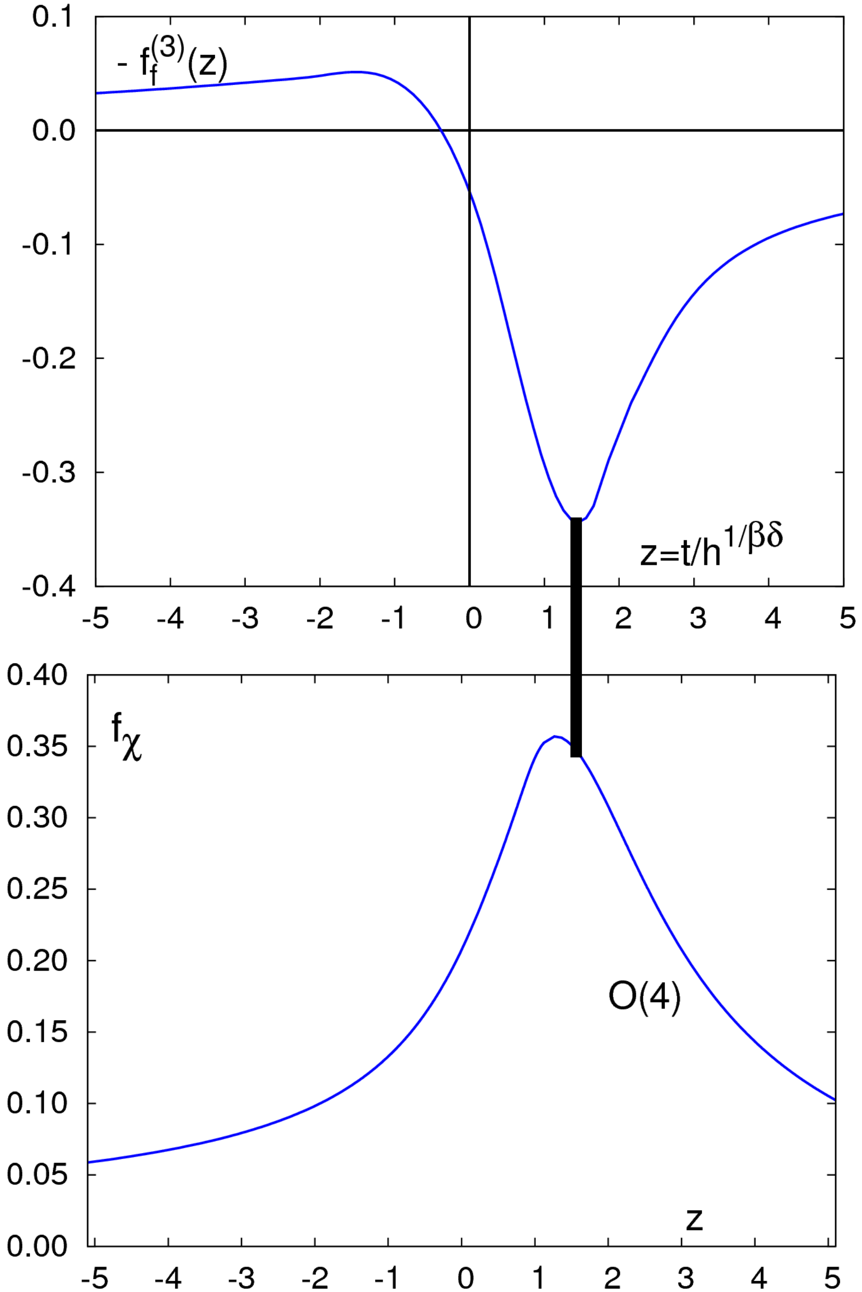
for $m_l/m_s = 1/27$

$$\frac{\chi_{6,0}^{B,min}}{\chi_{6,0}^{B,max}} = -6.7$$

chiral limit

universal number

Chiral crossover transition and 6th order cumulant at $\mu_B = 0$



quark mass scaling of transition temperature has been established

HotQCD, PR D85, 054503 (2012)

$$\frac{T_c(m_q) - T_c(0)}{T_c(0)} = \frac{1}{z_0 z_{max}} \left(\frac{m_l}{m_s}\right)^{1/\beta\delta} + reg.$$

$$z_{max} = 1.33(5)$$

$$z_+ \simeq 1.48$$

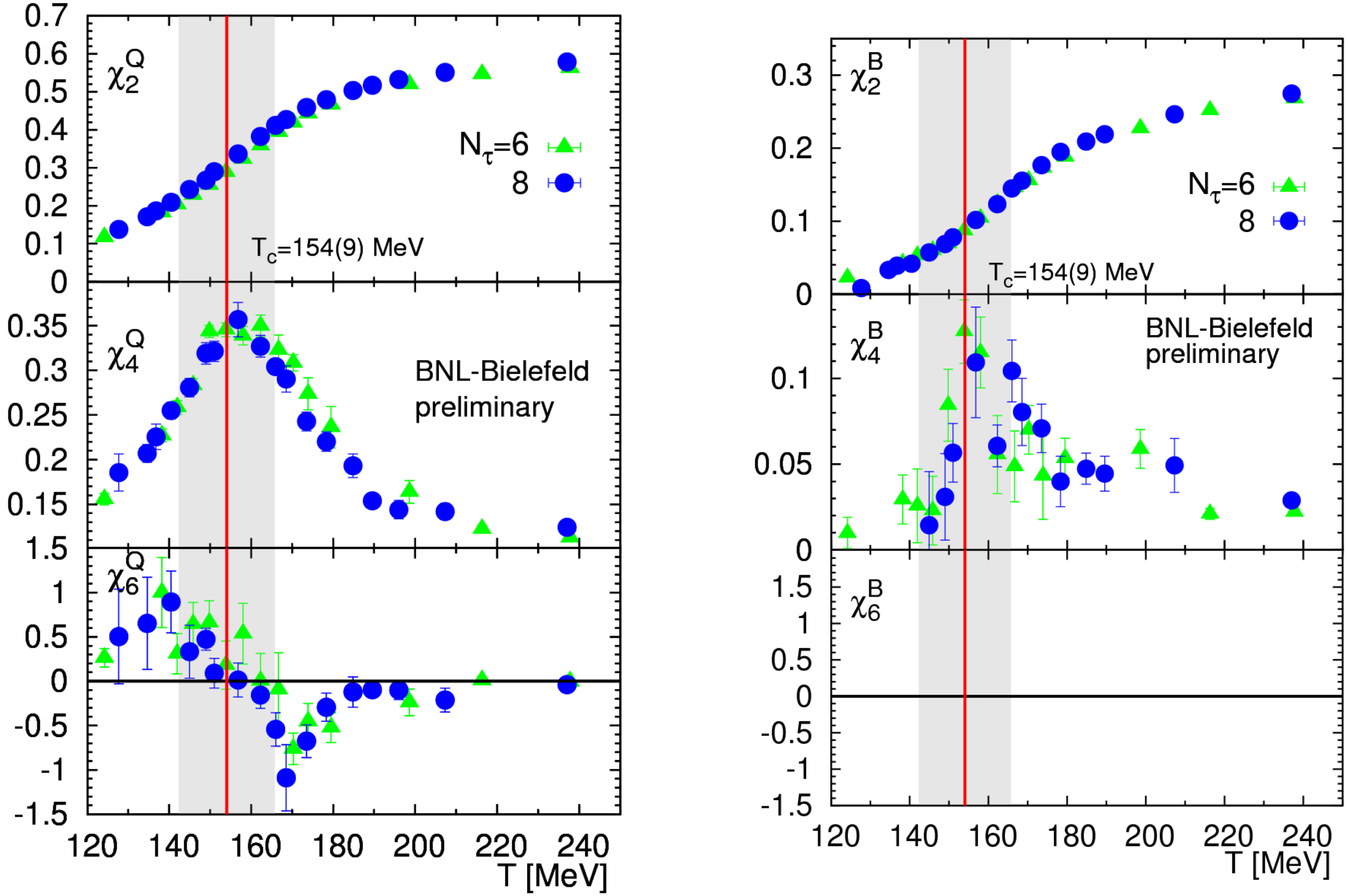
generically, i.e. when universal terms are dominant, the chiral crossover transition occurs in the region of negative $\chi_{6,0}^B$

contributions from regular part may change this

Electric charge fluctuations at the LHC

LHC: $\mu_B \simeq \mu_S \simeq \mu_Q \simeq 0$

- Cumulants calculated at $\mu_B = 0$ can directly be compared to (eventually available) data taken at LHC



$\boxed{\chi_6^Q \lesssim 0 \quad \chi_6^B \lesssim 0 \ soon}$

6th order cumulants differ strongly from HRG in the transition region

C. Schmidt (Bielefeld-BNL), Quark Matter 2012

Cumulants and critical behavior for $\mu_B > 0$

pressure: $$\frac{p}{T^4} = -h^{1+1/\delta} f_s(\underbrace{t/h^{1/\beta\delta}}_{z}) - f_r(V,T,\vec{\mu})$$

$$1+1/\delta = (2-\alpha)/\Delta \;,\; \Delta \equiv \beta\delta$$

$O(4):\ \alpha = -0.213$

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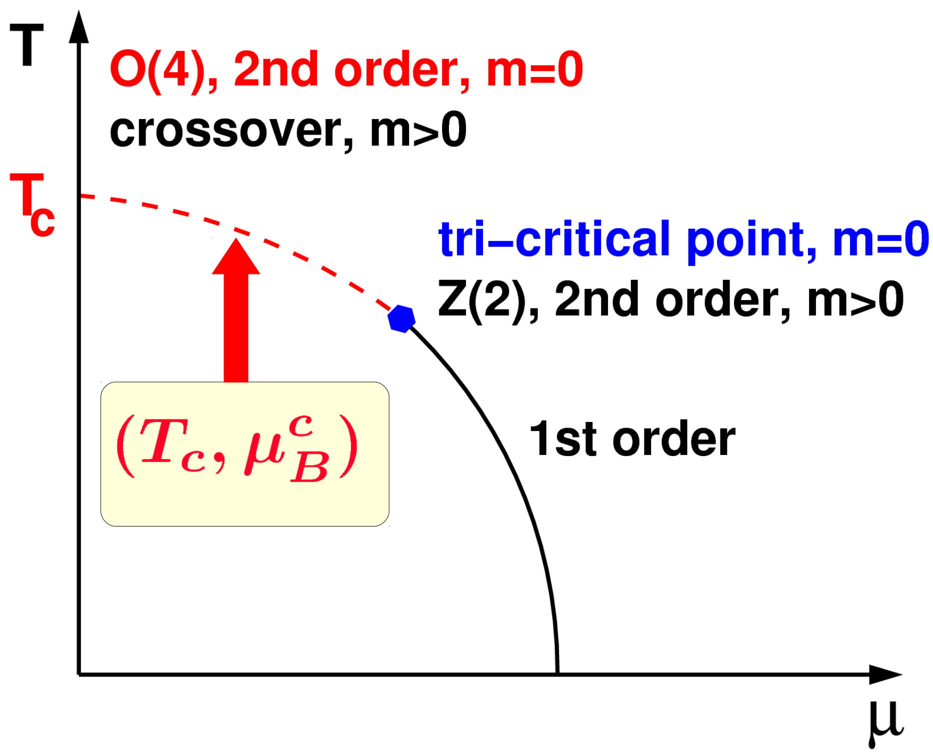


baryon number susceptibility:

$$\chi_2^B = \frac{\partial^2 p/T^4}{\partial(\mu_B/T)^2}$$

singular contribution at (T_c, μ_B^c)
starts generating a cusp with increasing μ_B^c

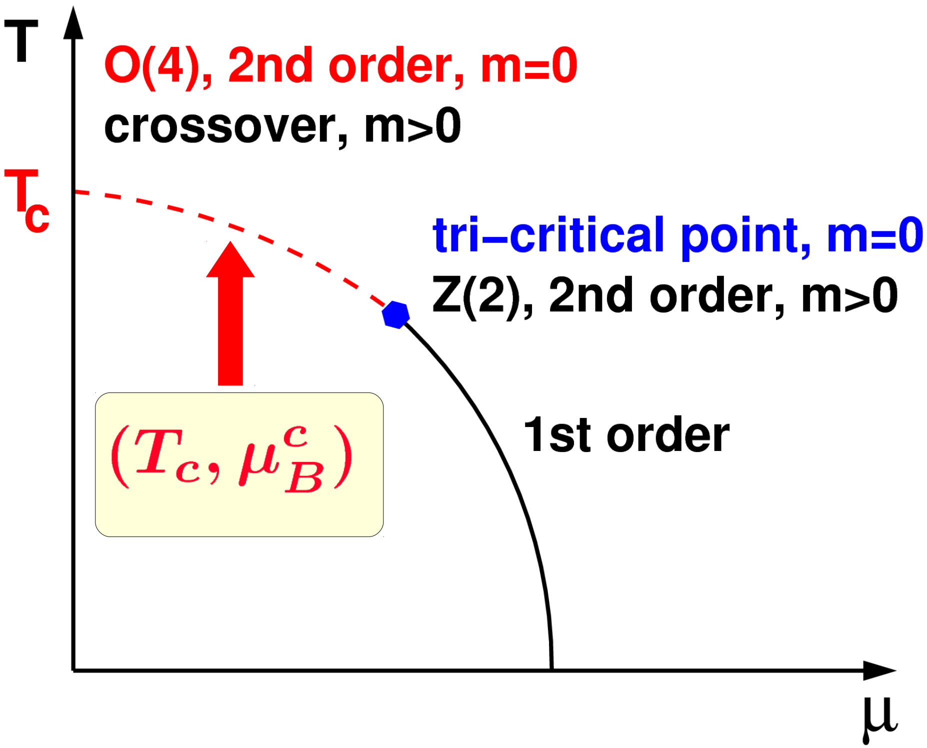
$$\boxed{\begin{aligned}\chi_2^B = &-2\kappa_B t_0^{-1} h^{(1-\alpha)/\Delta} f_s'(z)\\ &-(2\kappa_B t_0^{-1}\hat{\mu}_B^c)^2 h^{-\alpha/\Delta} f_s''(z)\end{aligned}}$$

Singular contribution to cumulants at $\mu_B > 0$

$$\chi_2^B = -2\kappa_B t_0^{-1} h^{(1-\alpha)/\Delta} \left(f_s'(z) + 2\kappa_B \underbrace{\frac{h_0^{1/\Delta}}{t_0} \frac{m_s}{m_l}}_{\mathcal{O}(1)} (\hat{\mu}_B^c)^2 f_s''(z) \right)$$

+ regular terms

$\mathcal{O}(1)$ for ms/ml=27



more singular terms gain importance

as $\hat{\mu}_B/T \gtrsim 1$ they generate a cusp

same combination of non-universal terms appears in all cumulants

$$z_0 = \frac{h_0^{1/\beta\delta}}{t_0} \simeq 2 - 3$$ Bi-BNL, preliminary hotQCD, 2012

$$\kappa_B = 0.0066(7)$$ Bi-BNL, 2011

need to control magnitude of non-universal parameters

4th order cumulant: A dip in the kurtosis at $\mu_B > 0$?

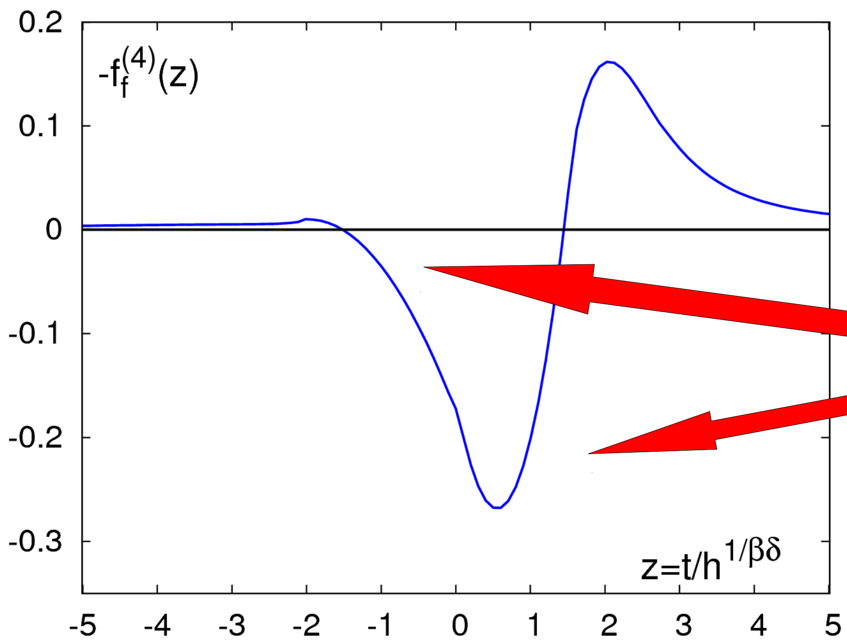
$$\mu_B > 0: \quad \chi^B_{4,\mu} = -3(2\kappa_B t_0^{-1})^2 h^{-\alpha/\Delta} f_s''(z)$$

$$-6(2\kappa_B t_0^{-1})^3 (\hat{\mu}^c_B)^2 h^{-(1+\alpha)/\Delta} f_s'''(z)$$

$$-(2\kappa_B t_0^{-1} \hat{\mu}^c_B)^4 h^{-(2+\alpha)/\Delta} f_s^{(4)}(z) \quad \text{+ regular}$$

dominates in the chiral limit, or if

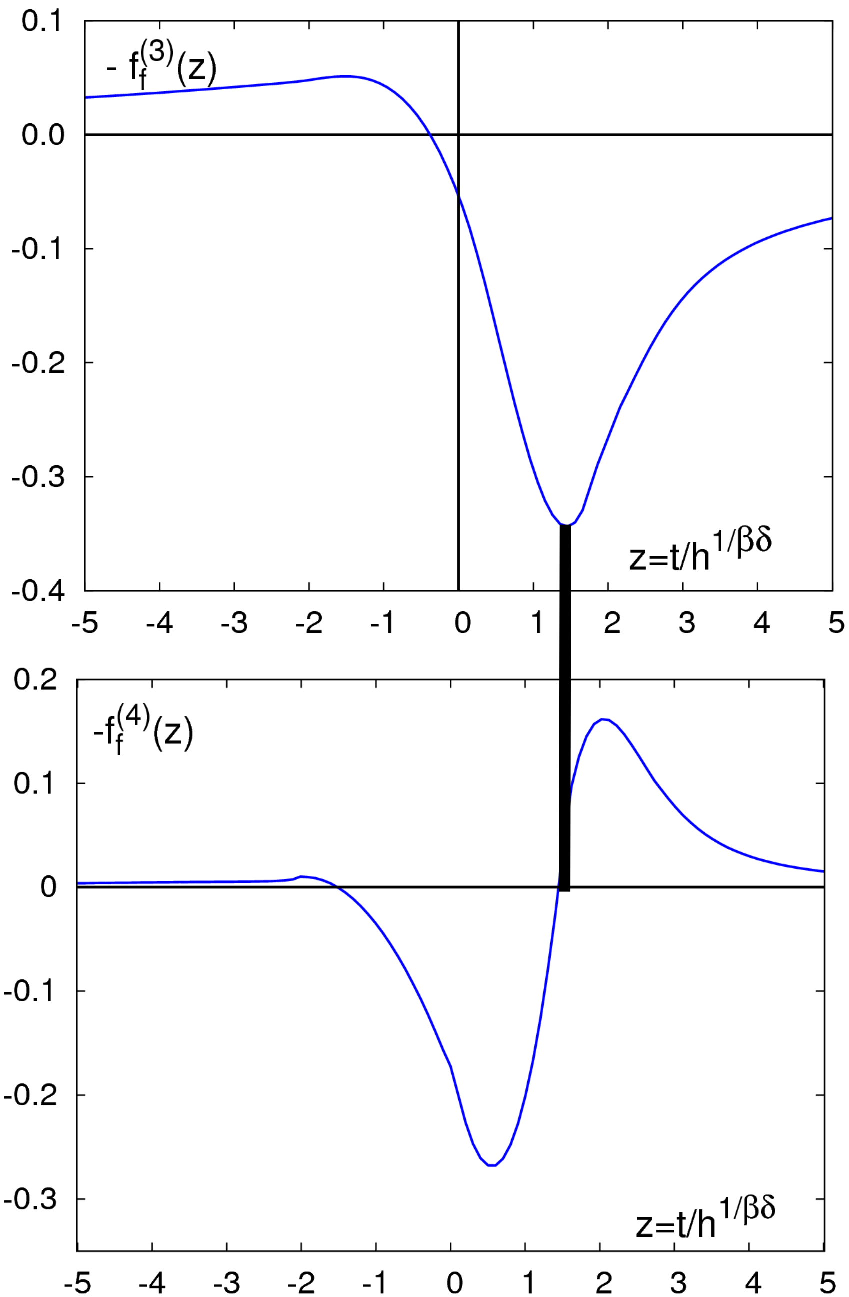
$$\mu^c_B/T \gtrsim 1$$



$$\frac{\chi^{B,min}_{4,\mu}}{\chi^{max-}_{4,\mu}} \simeq -25$$

B.Friman, FK, K.Redlich,V.Skokov,
Eur. Phys. J. C71, 1694 (2011)

4^{th} order cumulant (kurtosis) is negative for $T(\mu_B^c) \gtrsim T_{CP}$



I) at the crossover transition, $z = z_{max}$
$f_f^{(3)}(z)$ dominates $\Rightarrow$ $\chi_{4,\mu}^B < 0$

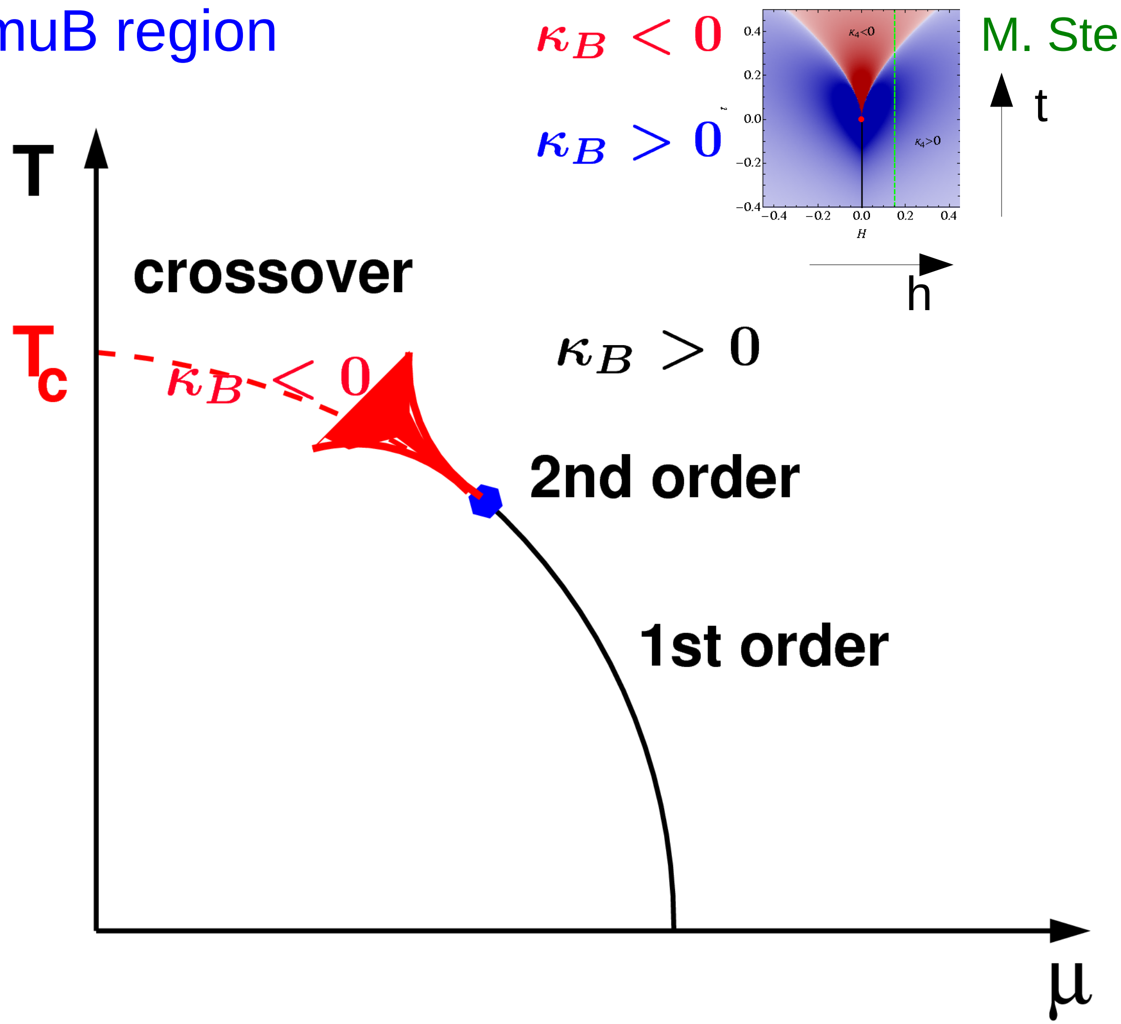
II) if $z^{freeze} < z^{crossover}$
$\Rightarrow$ $\chi_{4,\mu}^B < 0$ below but close to the chiral cross-over line

contributions from regular part may change this

4th order cumulant (kurtosis) and the critical point

in the vicinity of the critical point the kurtosis will be negative in a certain T, muB region

M. Stephanov, PRL 107, 052301 (2011)



mapping of the Ising variables t, h on the T, μ_B plane is non-trivial

4^{th} order cumulant (kurtosis) and the critical point

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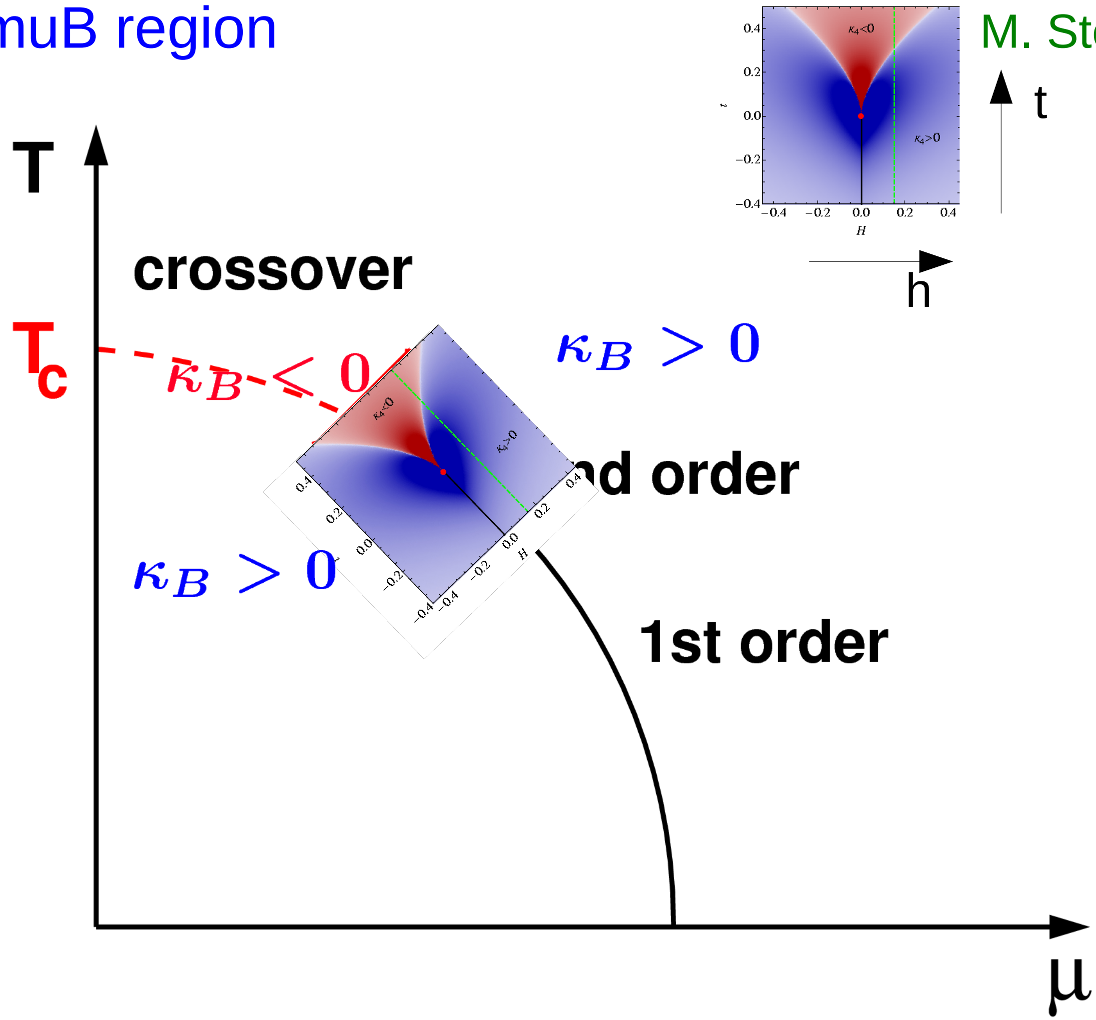
M. Stephanov, PRL 107, 052301 (2011)



mapping of the Ising variables t, h on the T, μ_B plane is non-trivial

4^{th} order cumulant (kurtosis) and the critical point

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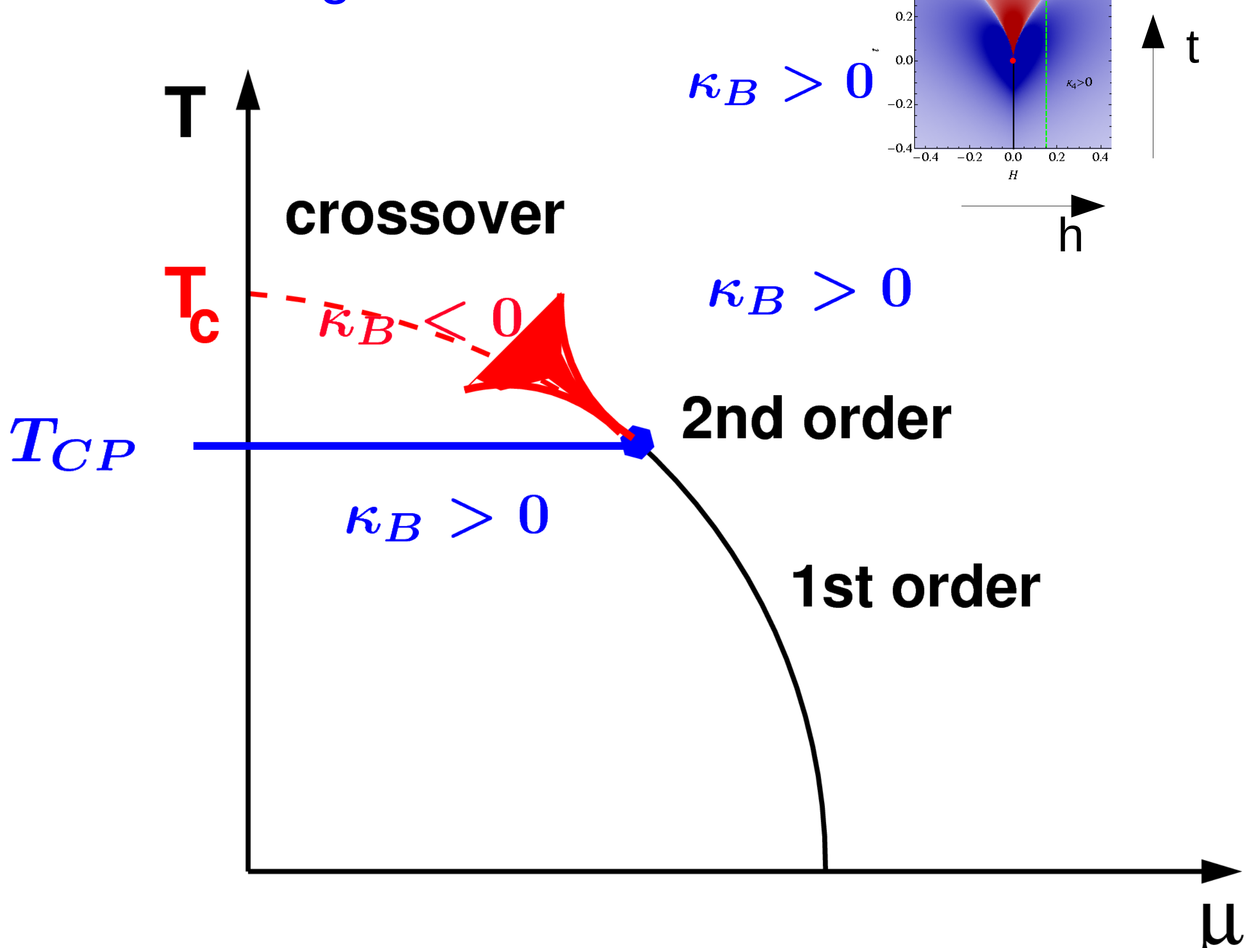
M. Stephanov, PRL 107, 052301 (2011)



mapping of the Ising variables t, h on the T, μ_B plane is non-trivial

generically, expect:

$\kappa_B > 0$ for $T \le T_{CP}$

expect all cumulants to be positive on the line of fixed $T \equiv T_{CP}$

prerequisite for well-behaved estimates of the location of the critical point based on the radius of convergence of the Taylor series for $\chi_{B,\mu}$

4th order cumulant (kurtosis) is positive at $T(\mu_B) \equiv T_{CP}$

most likely

- if the first order transition line is "steep", the "universal Z(2) cone" will open in the direction $T > T_{CP}$

- if the critical point exists, all expansion coefficients in a Taylor series of the pressure will be positive for some $n > n_0$

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} \frac{1}{(2n)!} \chi_{B,0}^{(2n)}(T) \left(\frac{\mu_B}{T} \right)^{2n}$$

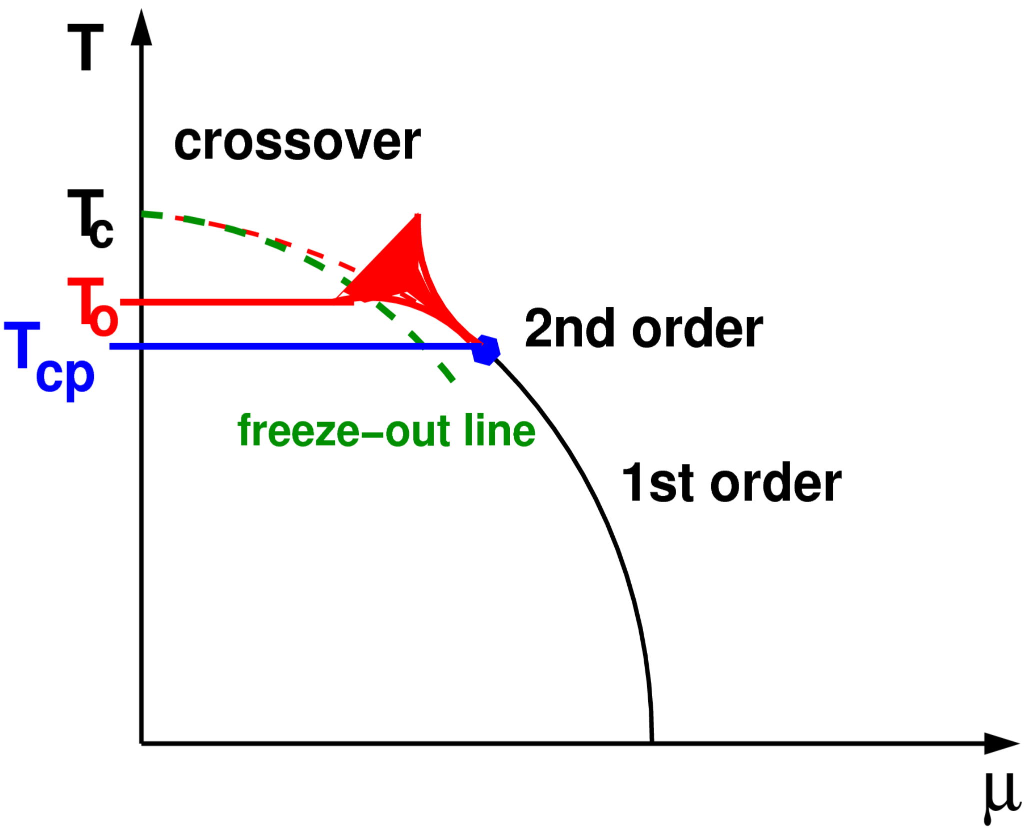
current indications are that ALL expansion coefficients of the pressure are positive at $T = T_{CP}$

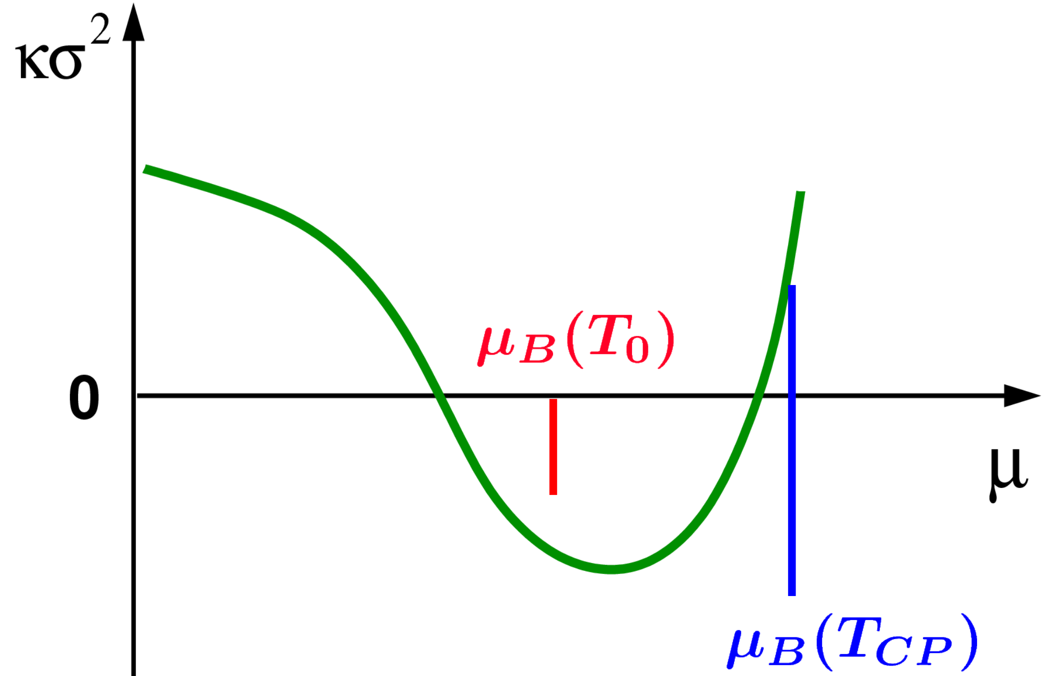
$$\Rightarrow \quad \kappa_B(T_{CP}, \mu_B < \mu_{CP}) \sim \left. \frac{\partial^4 p/T^4}{\partial(\mu_B/T)^4} \right|_{T=T_{CP}} > 0$$

B.Friman, FK, K.Redlich,V.Skokov, Eur. Phys. J. C71, 1694 (2011)

NB: If this is not correct, a determination of the critical point from the radius of convergence of the Taylor series will not work out !!!

Kurtosis on the freeze-out curve

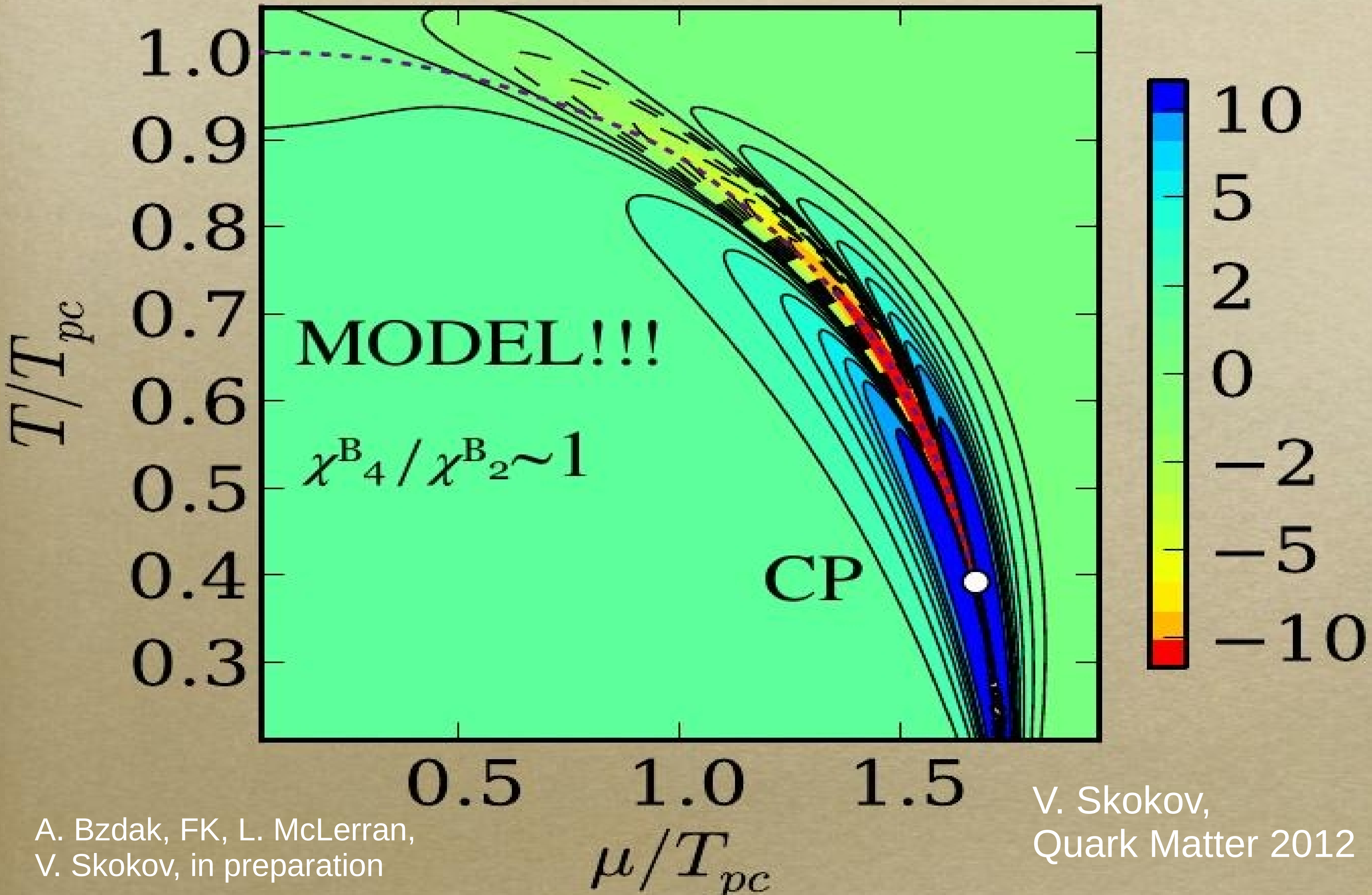




to determine the importance of regular terms and the non-universal scales requires lattice QCD

a dip in the kurtosis seems to be generic: whether or not it becomes negative depends on the magnitude of regular terms in the QCD partition function (pressure)

Chiral model and negative $\chi^B{}_4/\chi^B{}_2$:

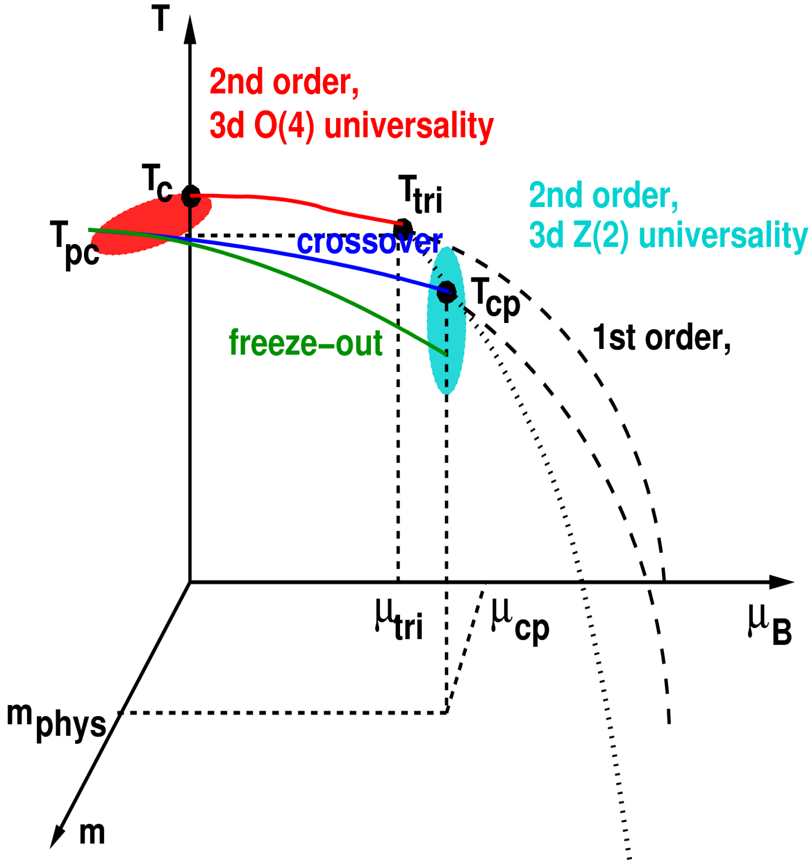


A. Bzdak, FK, L. McLerran, V. Skokov, in preparation

V. Skokov, Quark Matter 2012

Phase diagram for $\mu_B \geq 0,\ m_q > 0$

Does freeze-out occur close to a critical point?



- critical line at m_q=0

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_B \left(\frac{\mu_B}{T}\right)^2 - \mathcal{O}(\mu_B^4)$$

- crossover line: physics on crossover line controlled by universal scaling relations ?
  - freeze-out line: Is the crossover line related to the experimentally determined freeze-out curve?
  - net charge fluctuations: Do they probe thermodynamics on the freeze-out line?

→ quantitative questions that require lattice QCD to be answered

Taylor expansion of conserved charge fluctuations

$$\chi_{n,\mu}^{Q} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{i(j+n)k}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$\underline{X = B,\ Q,\ S}$

(similar for B and S)

$$\chi_{1,\mu}^{X} = \frac{1}{VT^3}\langle N_X\rangle$$

mean $M_X = VT^3\chi_{1,\mu}^X$

$$\chi_{2,\mu}^{X} = \frac{1}{VT^3}\langle(\delta N_X)^2\rangle$$

variance $\sigma_X^2 = VT^3\chi_{2,\mu}^X$

$$\chi_{3,\mu}^{X} = \frac{1}{VT^3}\langle(\delta N_X)^3\rangle$$

skewness $S_X\sigma_X = \frac{\chi_{3,\mu}^X}{\chi_{2,\mu}^X}$

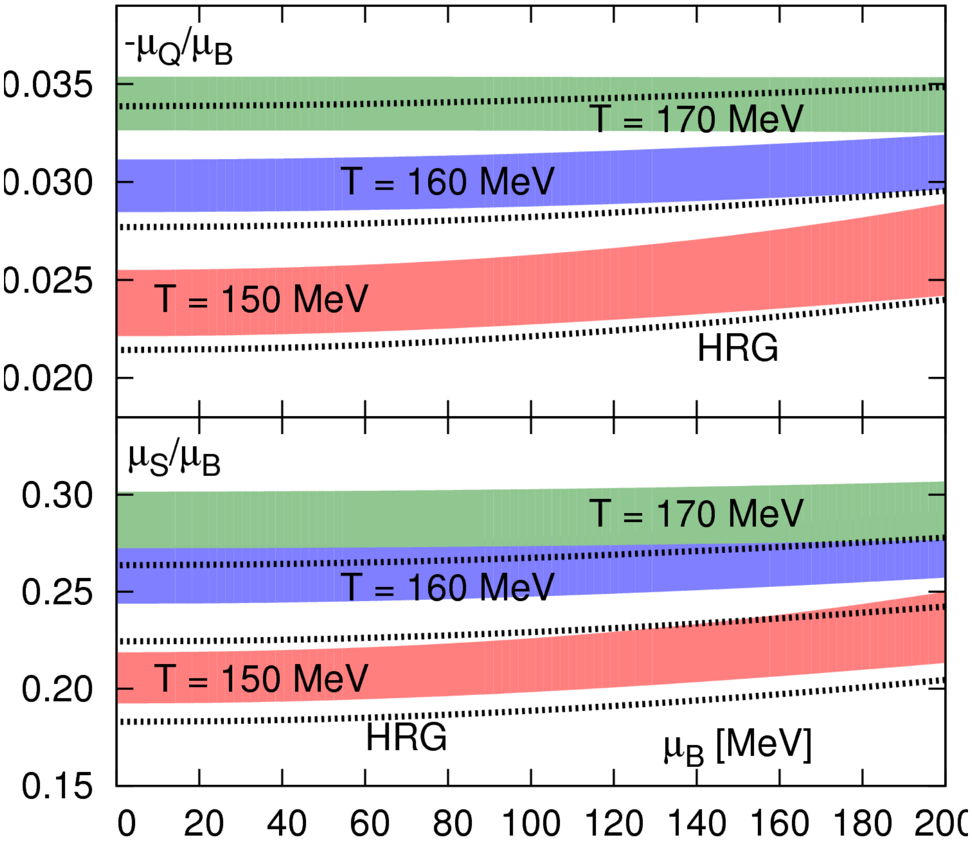
$$\chi_{4,\mu}^{X} = \frac{1}{VT^3}\left(\langle(\delta N_X)^4\rangle - 3\langle(\delta N_X)^2\rangle^2\right)$$

kurtosis $\kappa_X\sigma_X^2 = \frac{\chi_{4,\mu}^X}{\chi_{2,\mu}^X}$

e.g. take ratios to
eliminate volume factor

Strangeness and electric charge chemical potentials: Next to Leading Order (NLO) results at fixed T

for 150MeV < T < 170MeV QCD and HRG agree within ~10% on μ_S/μ_B , μ_Q/μ_B



NLO Taylor expansions for electric charge and strangeness chemical potentials are well behaved for

$$\mu_B/T \lesssim 1.3$$

for orientation: $\mu_B = 1.3T \Leftrightarrow$

$$\mu_B = 200\ \mathrm{MeV\ at}\ T = 160\ \mathrm{MeV}$$

tempting to compare with STAR result (QM'12),

$$\frac{\mu_S}{\mu_B} \simeq (0.2 - 0.25)$$

However,..

this covers RHIC experiments down to

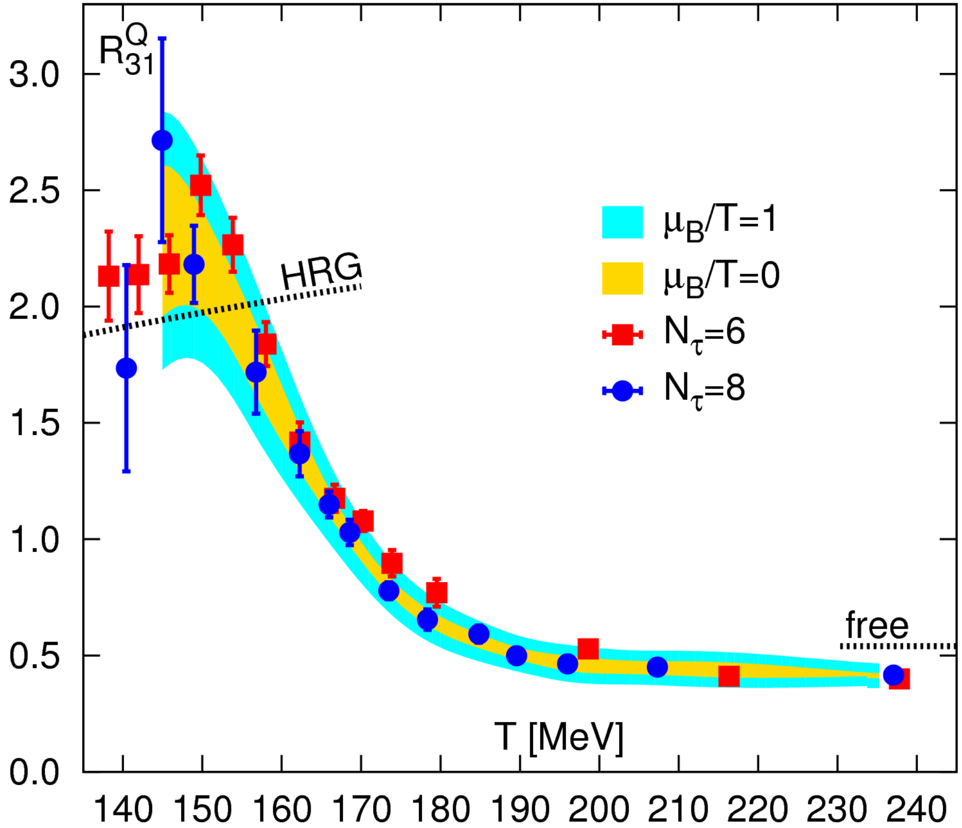
$$\sqrt{s_{NN}} \simeq 20\ \mathrm{GeV}$$

Bielefeld-BNL, arXiv:1208.1220, PRL to appear

Fixing T with a Thermo-meter: R_{31}^{X} , $X = Q, B$

$$R_{31}^{X} \equiv \frac{S_X \sigma_X^3}{M_X} = R_{31}^{X,0} + R_{31}^{X,2}\ \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$R_{31}^{Q,0} = \frac{\chi_{13}^{BQ} + q_1 \chi_4^{Q} + s_1 \chi_{31}^{QS}}{q_1 \chi_2^{Q} + \chi_{11}^{BQ} + s_1 \chi_{11}^{QS}}$$



$R_{31}^{Q,2}$ requires 6[th] order coefficients (estimate of its magnitude) NLO corrections below 10%

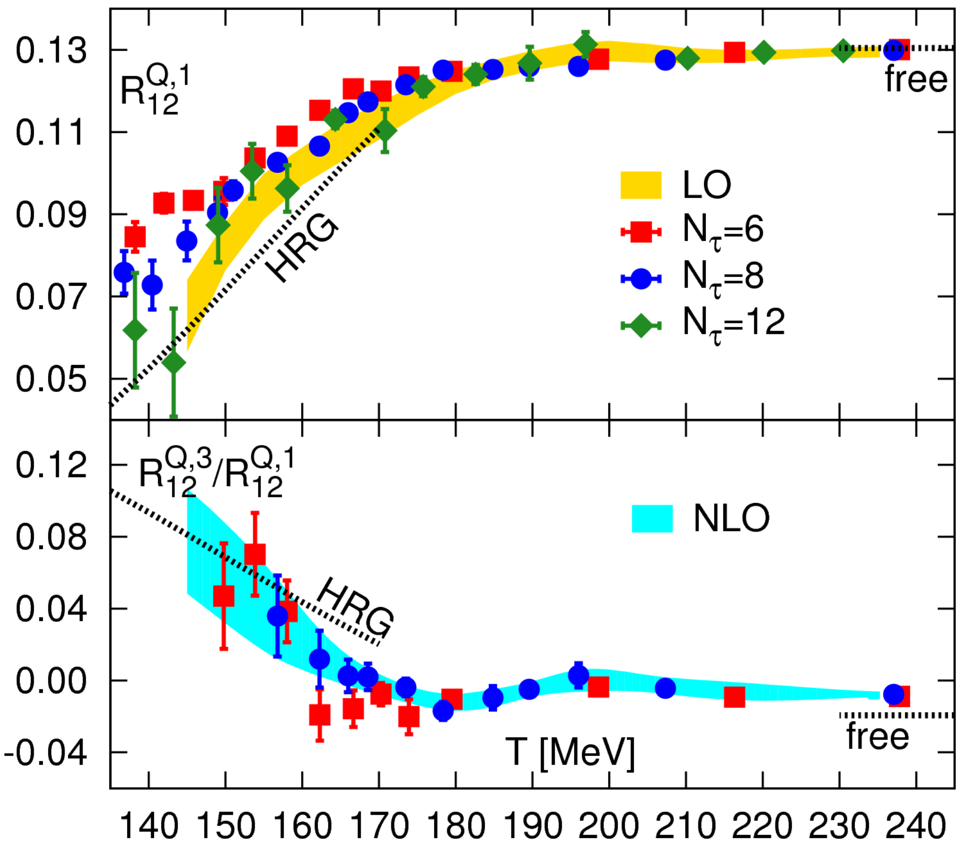
$R_{31}^{Q,0}$ provides stringent constraint on T

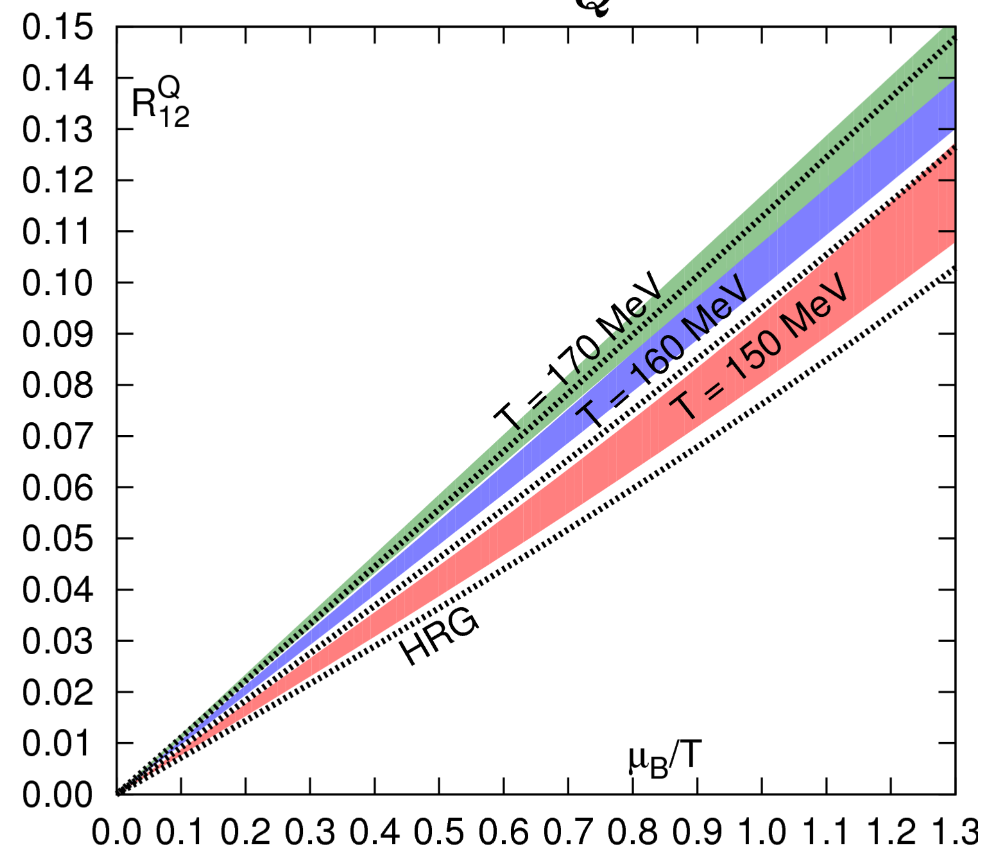
large deviations from HRG for T>155 MeV

Fixing μ_B with a Baryo-meter: R_{12}^{X} , $X = Q, B$

R_{12}^{Q} provides stringent constraint on μ_B/T

$$R_{12}^{Q} = \frac{M_Q}{\sigma_Q^2}$$



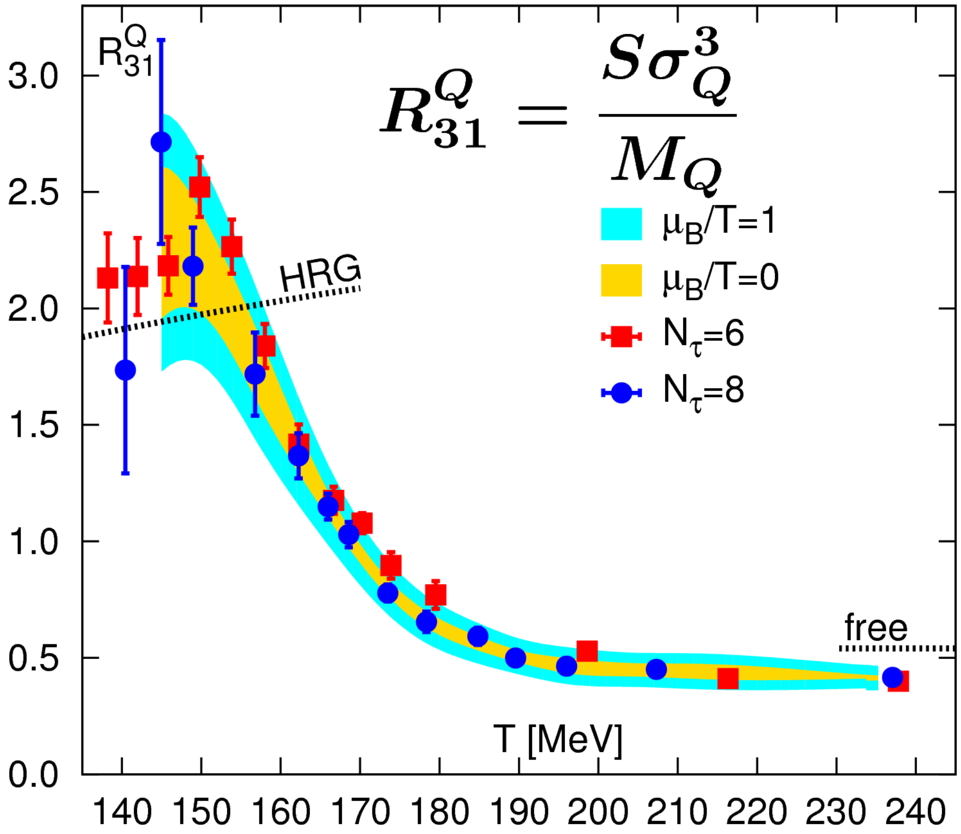


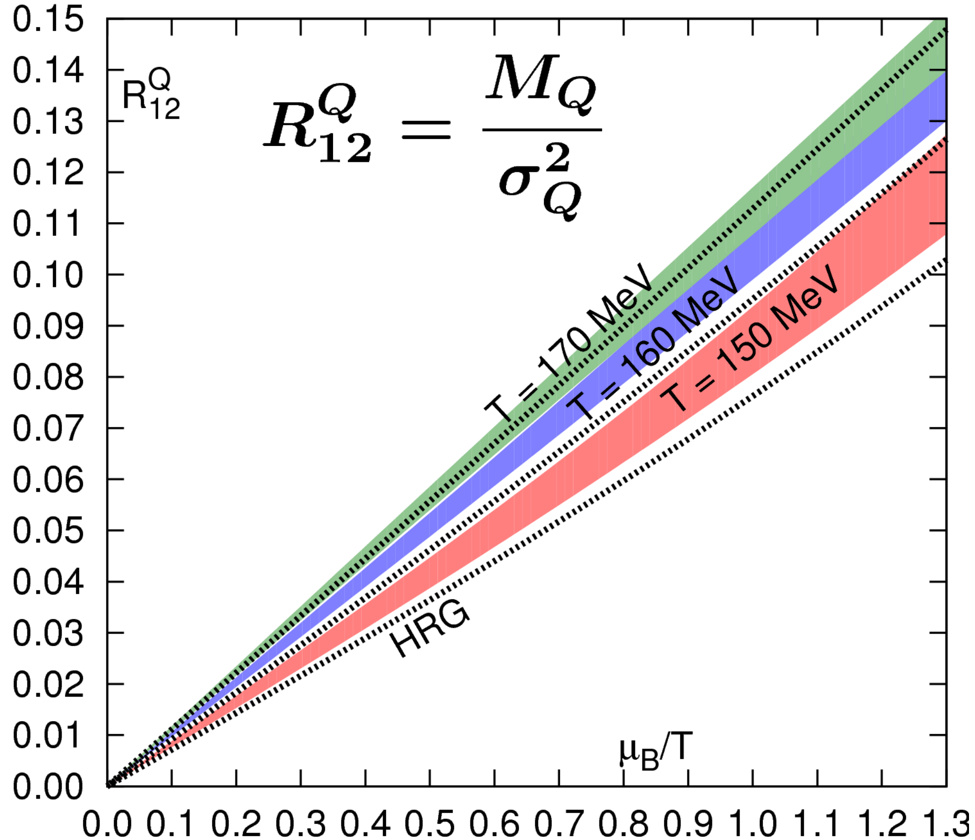
Bielefeld-BNL, arXiv:1208.1220, PRL to appear

LO: continuum extrapolated
NLO: spline interpolation

NLO correction contributes less than 10% for T>140MeV and $\mu_B/T \leq 1$

Determination of T and μ_B





Bielefeld-BNL, arXiv:1208.1220, PRL to appear

need data for these two observables to determine

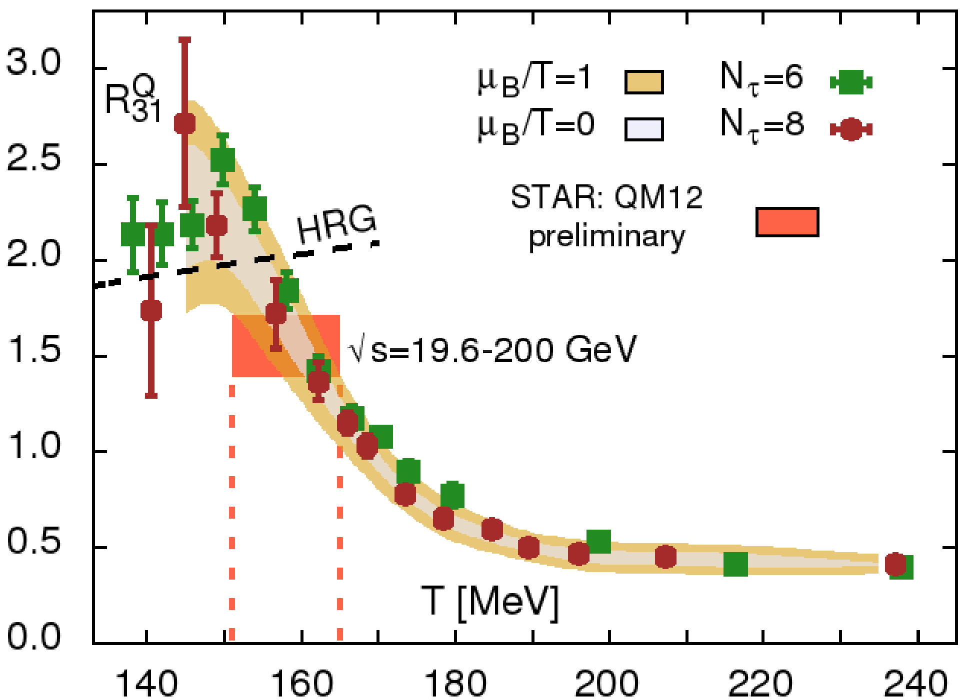
$$T,\ \mu_B$$

from then on all other cumulant ratios probe thermodynamic consistency, i.e. our basic assumption of a unique freeze-out line, equilibrium thermodyn., etc

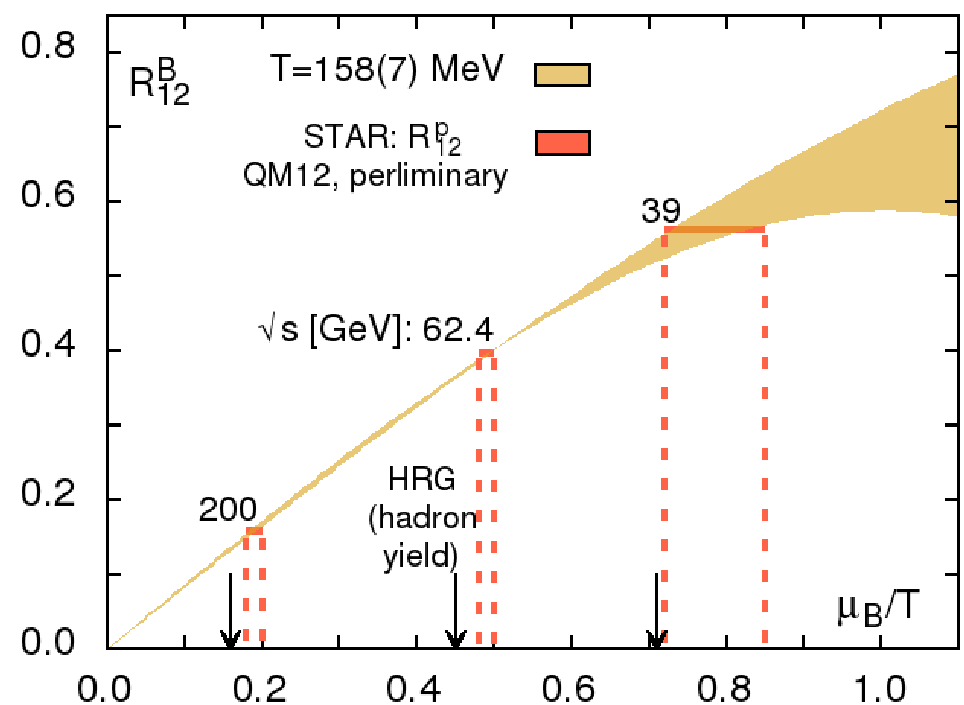
Determination of T and μ_B

$$R_{31}^{Q} = \frac{S\sigma_Q^3}{M_Q}$$

$$R_{12}^{p} = \frac{M_p}{\sigma_p^2}$$



T=158(7) MeV



similar to HRG, but
$R_{12}^p = R_{12}^B$??
and what about...?

from then on all other cumulant ratios probe thermodynamic consistency, i.e. our basic assumption of a unique freeze-out line, equilibrium thermodyn., etc

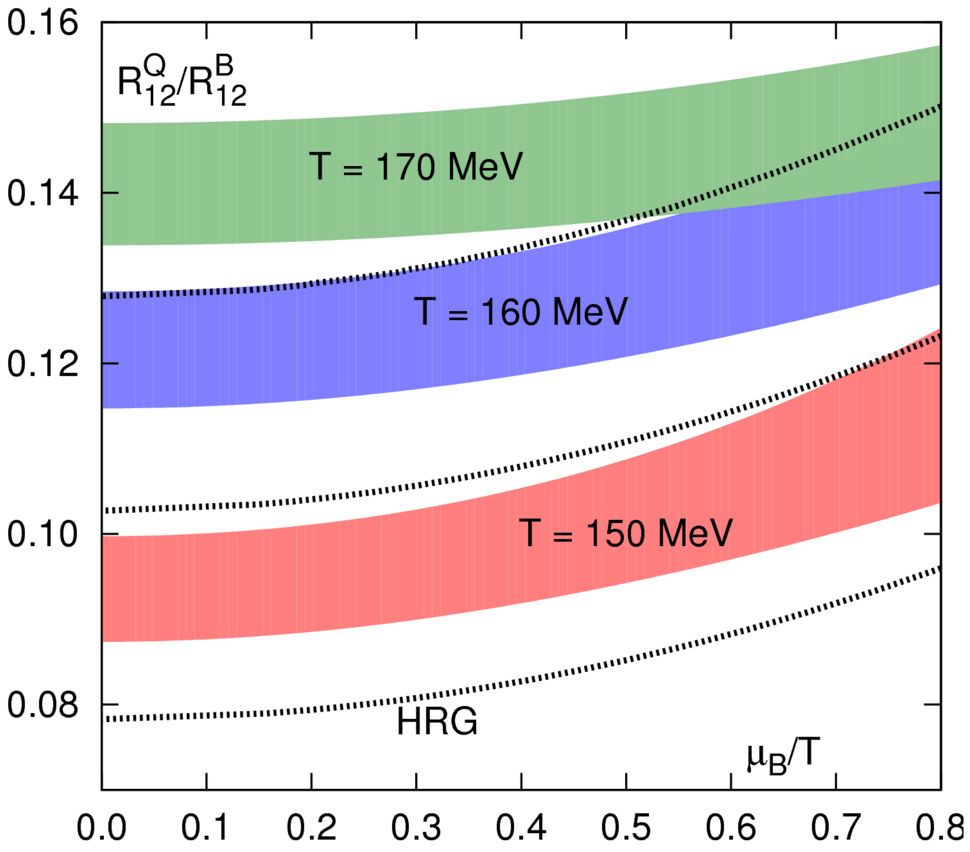
Thermodynamic consistency

$$R_{12}^Q = \frac{M_Q}{\sigma_Q^2} \quad \text{and} \quad R_{12}^B = \frac{M_B}{\sigma_B^2}$$

should provide identical information on T and μ_B

ratio of variances of electric charge and baryon number fluctuations

$$R_{QB} = \frac{R_{12}^Q}{R_{12}^B} = r\frac{\chi_2^B}{\chi_2^Q}$$



experimentally only net proton rather than net baryon number fluctuations are accessible

$$R_{nm}^B = R_{nm}^p \quad ???$$

STAR preliminary at 200 GeV:

$$\frac{R_{12}^Q}{R_{12}^{proton}} \simeq 0.06$$

a problem!!

Conclusions

– higher order cumulants of net charge fluctuations are very promising observables to search for critical behavior and to make contact between (lattice) QCD and HIC experiments.

– higher order cumulants will be quite different from HRG model calculations in the transition region

– the relative magnitude of singular and regular terms cannot be determined in model calculations but requires a QCD calculation

– through a comparison of equilibrium QCD calculations with HIC data on cumulants up to 6^{th} order it soon will become possible to test whether fluctuations of conserved charges can consistently be described by equilibrium thermodynamics with a unique set of freeze-out parameters.

wait for S. Mukherjee's talk tomorrow